Advanced Music Theory

For MUSC 320 at Manhattan College, Spring 2024

Version 0.75 created 4/19/24 by Dave Smey

Introduction

Welcome to our advanced theory text, which I'll build in bits and pieces as the semester rolls on. For this semester we *really* need a custom textbook because we'll be sampling lots of different topics, trying to hit as much as we can. There's no one book that does it all!

As before I'm going to encourage you to invest in some more professional textbooks if you are really interested in this stuff. For the more traditional classical theory topics we'll do first, I recommend:

> Kostka, Stefan M., and Byron Almén. *Tonal Harmony*. 9th ed. McGraw-Hill, 2023.

> Aldwell, Edward, Carl Schachter, and Allen Cadwallader. *Harmony and Voice Leading*. 5th ed. Cengage Learning, 2018.

(I'm citing the most recent editions, but the smart move is usually to look for an older copy which will be much cheaper. Old editions of most textbooks are perfectly good, but academic publishers revise them in order to disrupt the used book market.)

The "serious" study of rock harmony is actually fairly new, so the work being done on it is still truly theoretical. If one is up for a thought-provoking volume you could try

> Doll, Christopher. *Hearing Harmony: Toward a Tonal Theory for the Rock Era*. Ann Arbor: University of Michigan Press, 2017.

Finally, for Modern and "post-tonal" theory I like

> Rahn, John. *Basic Atonal Theory.* New York: Schirmer, 1980.

(That's very much out of print, try interlibrary loan), or

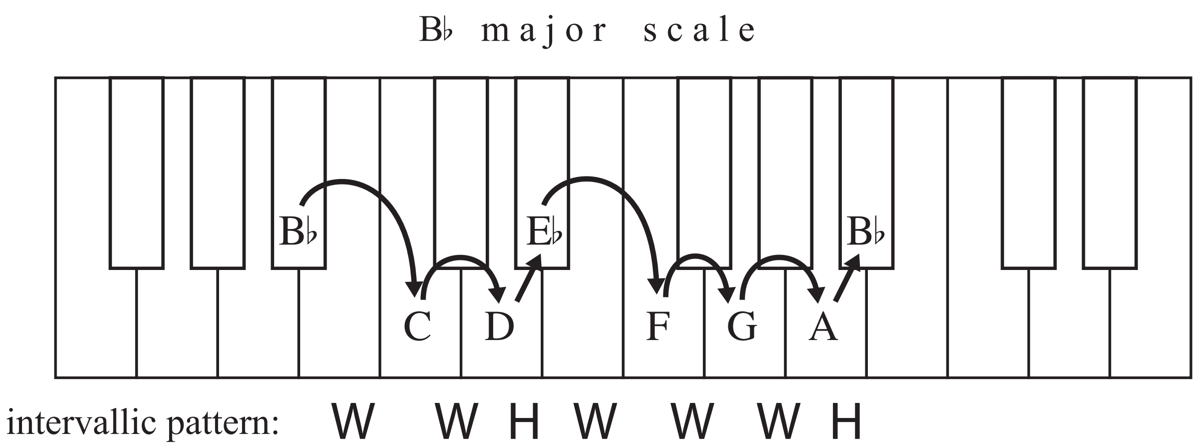
> Straus, Joseph N. *Introduction to Post-Tonal Theory.* 4th ed. W. W. Norton & Co., 2016.

Chapter 1: The Basic Elements from Theory I

Here are some diagrams that illustrate basic ideas we learned in Theory I. Remember that if you are still fuzzy on these concepts you can always surf into the MUSC220 blog and look at the Theory I textbook.

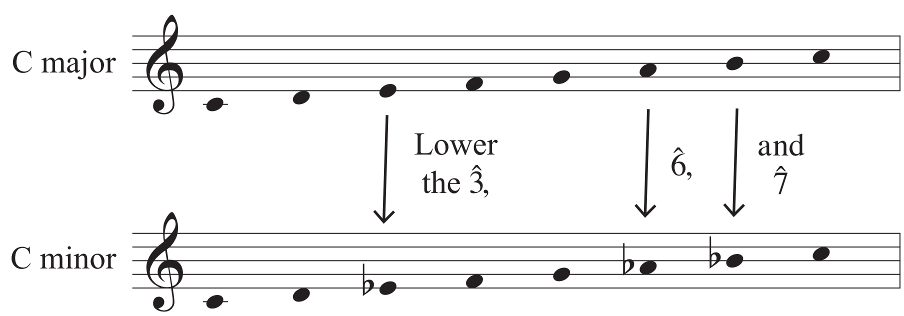
Major Scales

Understanding major scales as WWHWWWH.



Minor Scales

We learned minor as a transformation of major...



...and we also talked about the relative minor / relative major relationship.

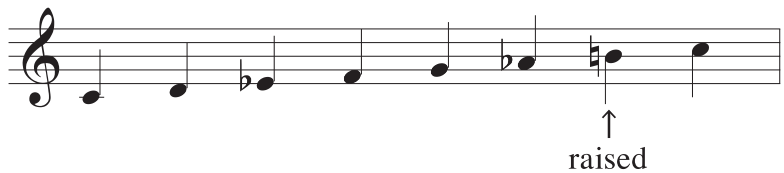
A minor	A B C D E F G A
C major	C D E F G A B C

Finding the relative minor.

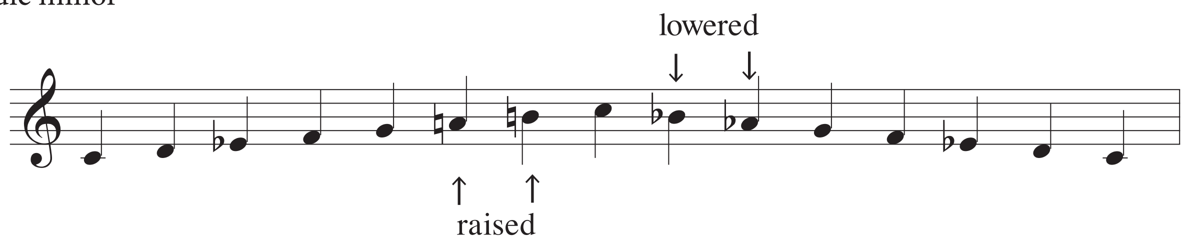
$$\begin{array}{cccccccc} & & & & & \hat{6} & \hat{7} & \hat{8} \\ \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{F} & \mathrm{G} & \mathrm{A} & \mathrm{B} & \mathrm{C} \\ & & & & & \text{relative minor} & & \end{array}$$

Variants of minor

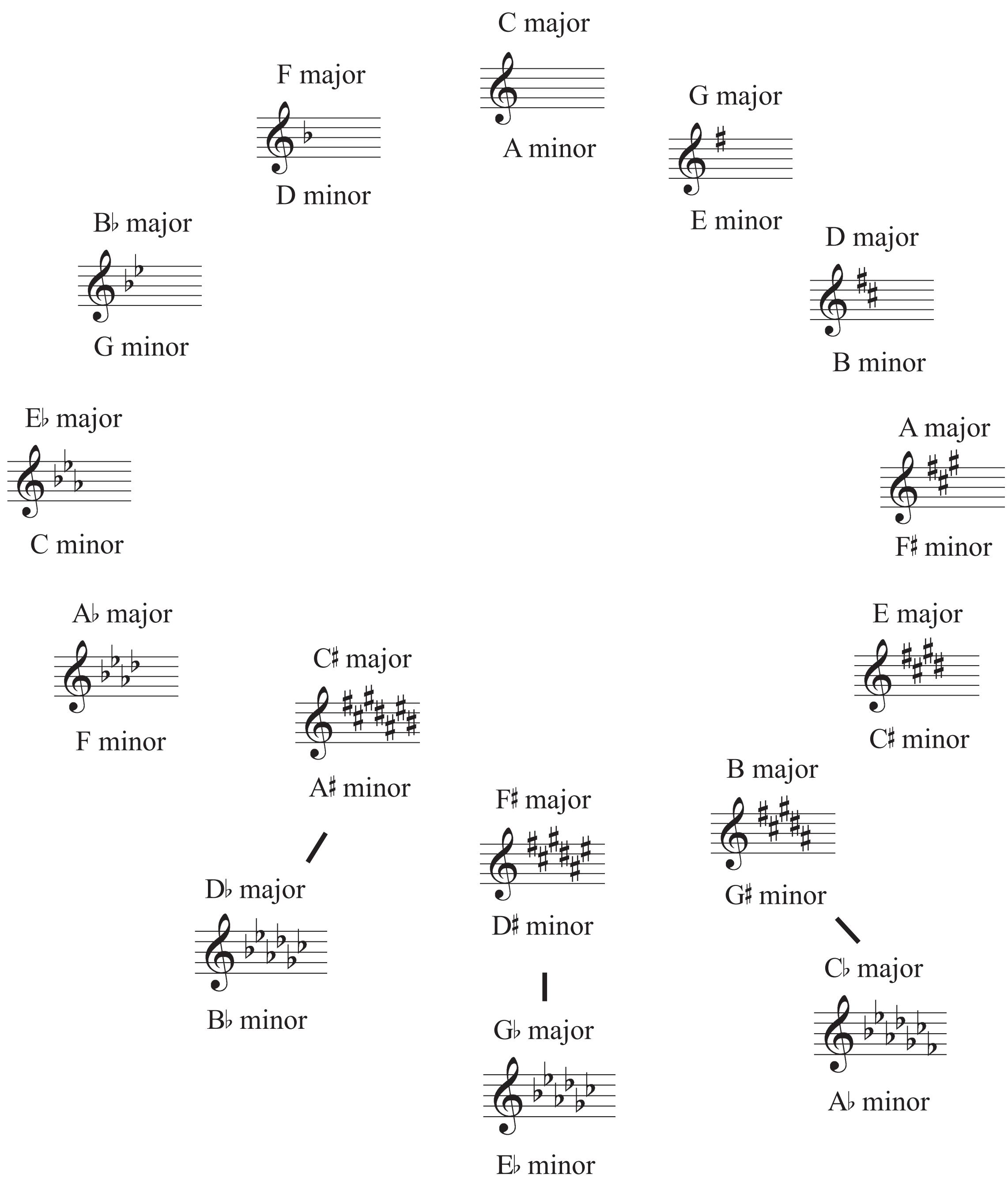
harmonic minor



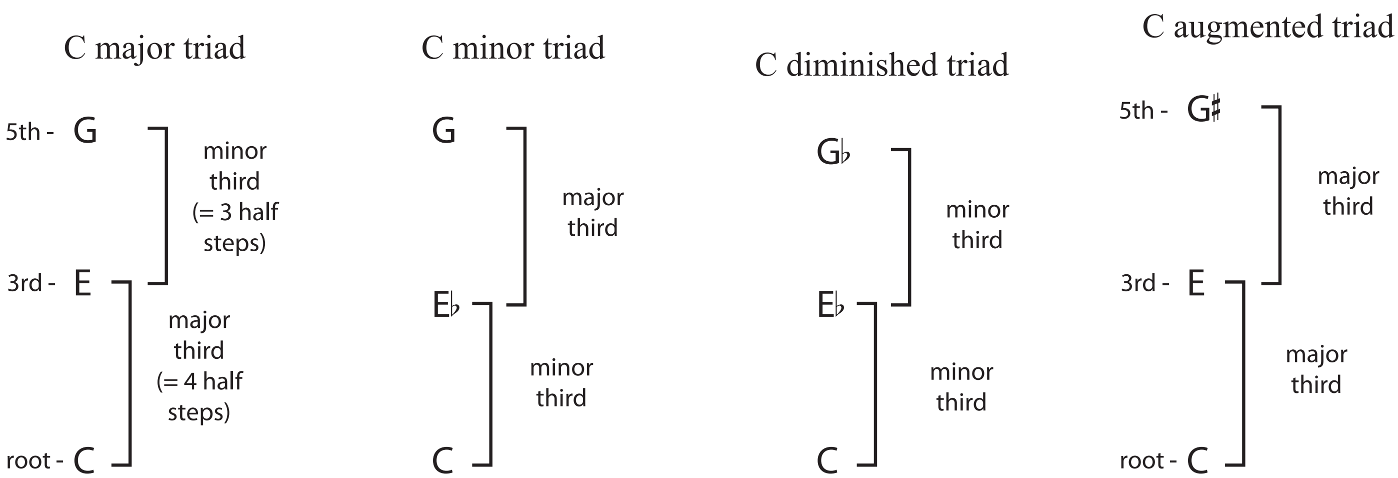
melodic minor



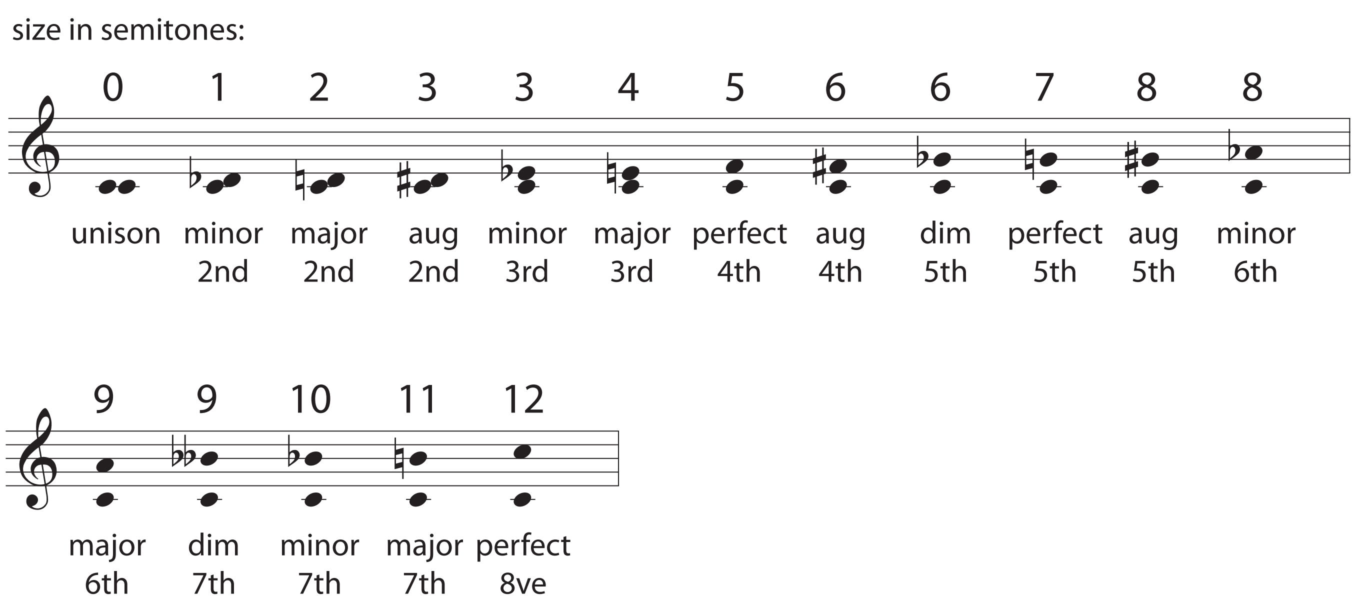
The circle of fifths



The four triad types



Intervals

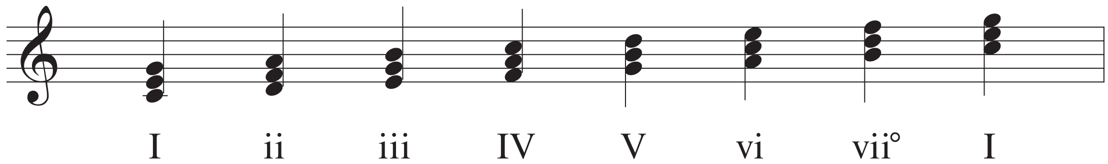


Unisons, fourth, fifths and octaves are the “perfect” intervals. They can be diminished, perfect, or augmented.

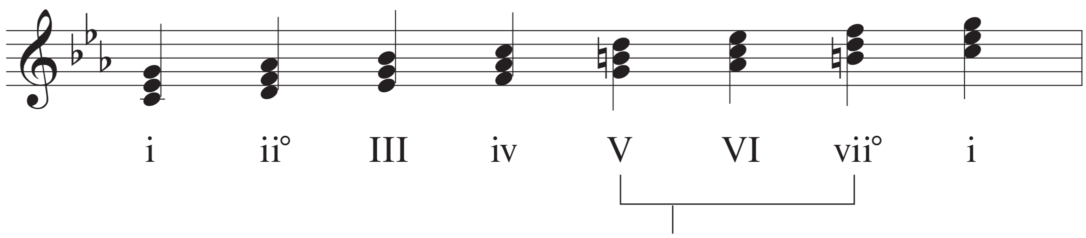
Seconds, thirds, sixths and sevenths are the “imperfect” intervals. They can be diminished, minor, major, or augmented.

Diatonic Triads (a.k.a. the Roman Numerals)

in C major



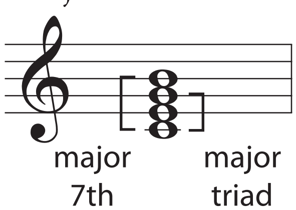
in C minor



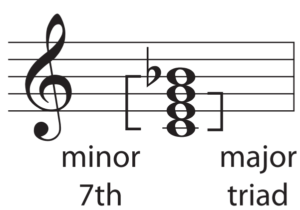
typical Classical-music alterations
with raised leading-tone

Seventh Chords

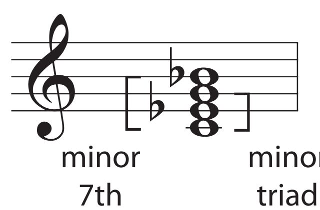
major seventh chord



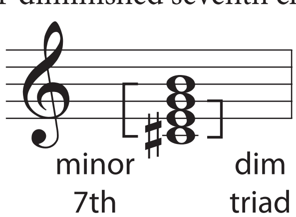
dominant seventh chord



minor seventh chord

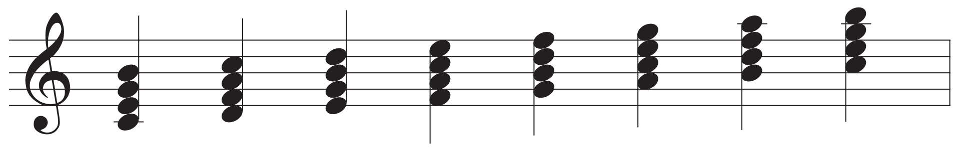


half-diminished seventh chord

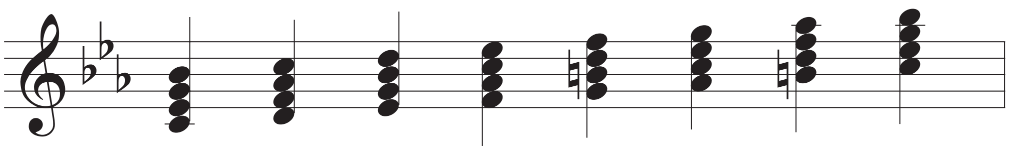


fully-diminished seventh chord

Diatonic Seventh Chords



C: I^{M7} ii^{7} iii^{7} IV^{M7} V^{7} vi^{7} $\mathrm{vii}^{ø7}$ I^{M7}

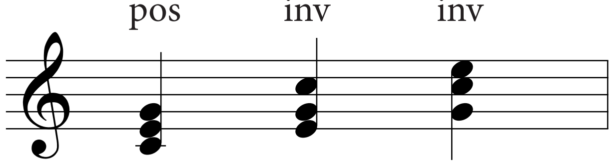


c: i^{7} $\mathrm{ii}^{ø7}$ III^{M7} iv^{7} V^{7} VI^{M7} $\mathrm{vii}^{\circ 7}$ i^{7}

Chord Inversions

triad inversion shapes

root pos | 1st inv | 2nd inv



$^{5}_{3}$ shape | $^{6}_{3}$ shape | $^{6}_{4}$ shape

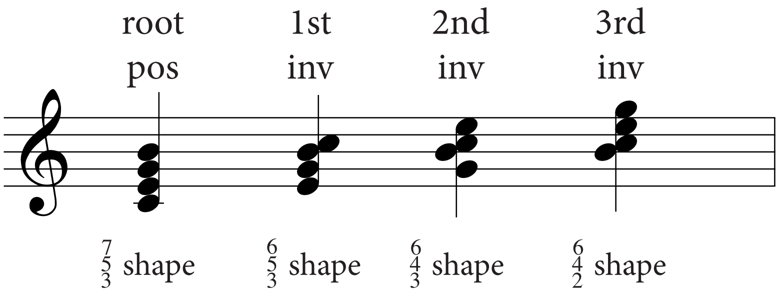
figured bass symbols for "real music"

root position = I

1st inversion = I^{6}

2nd inversion = I^{6}_{4}

seventh-chord inversion shapes



figured bass symbols for "real music"

root position = V^7

1st inversion = V^6_5

2nd inversion = V^4_3

3rd inversion = V^4_2

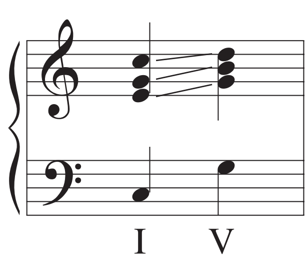
Writing Chord Progressions (with good counterpoint)

When the bass is moving by **fourth or fifth...**

Common-tone technique

One part holds over, the other two slide into place by step.

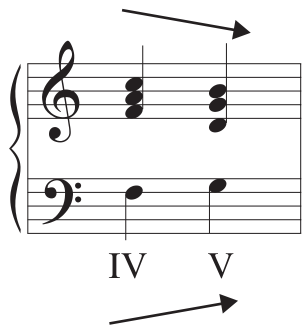
"Next-closest" technique



All parts move by a third or less.

When the bass is moving by **step...**

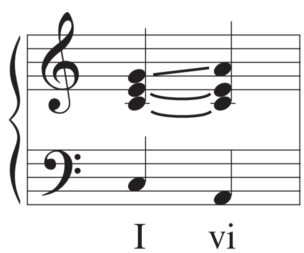
"Step Zone" technique



Slide to the next available shape in the opposite direction.

When the bass is moving by **third or sixth...**

Double common-tone technique



Hold over two notes, remaining tone moves by step.

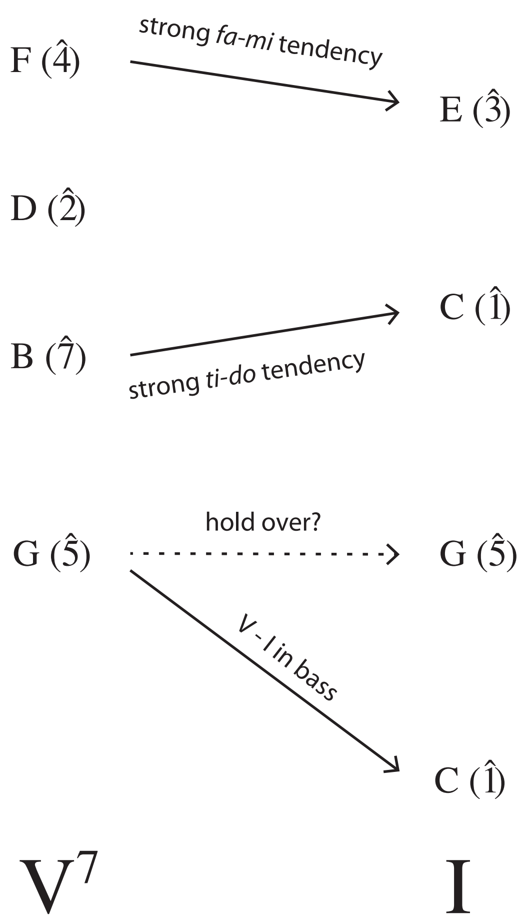
Chapter 2: Progressions with V7

The most common seventh chord in most kinds of music is V^7, which usually wants to resolve to I. In this chapter I will show you two quick formulas to write progressions with this slightly spicy harmony.

Tendency Tones

There is often a sense that the tones of a V^7 chord "want" to resolve in a certain way. This is probably caused by a combination of cultural and perceptual factors.

Let us consider a V7 chord in C major:



As it resolves, the leading-tone (*ti,* the third of the chord) "wants" to go up to *do*. The seventh of the chord (*fa*) also wants to resolve down to *mi*, especially in major keys. (There is something about the closeness of the target note, only a half-step away, that seems to make these resolutions particularly satisfying.)

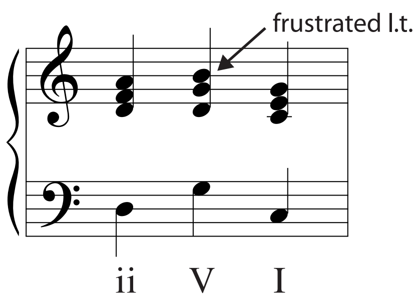
What the remaining tones want to do is more of a practical matter than a perceptually urgent one. The fifth of the chord (*re*) maybe has a weak tendency to go to *do*, but it could easily go up to *mi* if it needs to.

What you want to do with the root of the V7 (*sol*) depends on where it is in the texture. If it is your bass note, you probably want to jump to *do* to make the V-I bass line. However, if it is in an upper voice it can be a common tone that holds over into I.

The Frustrated Leading-Tone

Theorists are particularly concerned with the tendency of the leading tone, which wants to go from *ti* to *do*. If the leading tone is in the top line of your V or V^7 chord, it must go to *do* or else it is called a "frustrated leading tone."

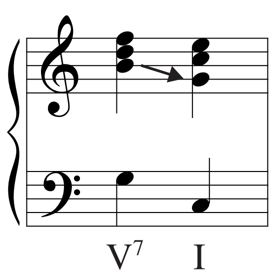
This is true even with a plain V chord! The idea is that leaping away from the leading tone is disappointing, because we anticipate the conventional resolution to *do* and will be annoyed if we don't get it.



A plain V chord with frustrated leading tone.

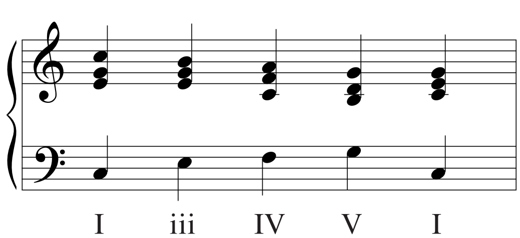
However, this rule is actually pretty narrow - it only applies when the leading tone is exposed in the top voice and you leap away from it instead.

If you hide the leading tone in an inner voice it's considered fine. This will actually be one of our strategies to make good V^7-I's.

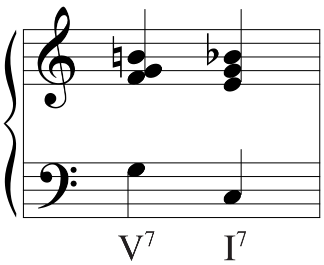


Frustrated leading tone in inner voice (which is fine)

There are also a few situations where the leading tone might connect downward by step that are not at all disappointing.



In this progression, the leading tone is part of the iii chord. It continues down by step and I don't think anybody would object. Stepwise connections are strong, so the motion from C-B-A is compelling and not disappointing in any way.

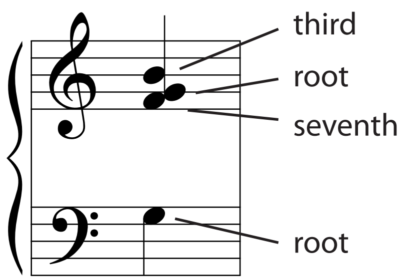


Similarly, in contexts where we go from one dominant seventh to another, the third of one chord often sinks down to the seventh of the next. Let us imagine that we are going from a V^7 to a I^7 (in some kind of bluesy context, perhaps). This is also fine.

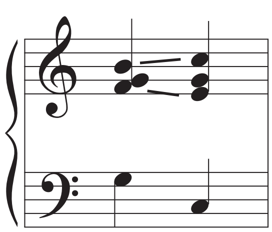
So with all of this tendency-tone business out of the way, let's look at two quick strategies to make good V^7-I progressions.

Strategy #1 - Omit the fifth, double the root

Surprisingly, you don't need to use all of the notes in your dominant seventh chord. You can omit the fifth, and nobody will miss it.

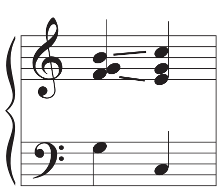


With this technique you build V^7 with the root on the bottom and root, third, and seventh on top.

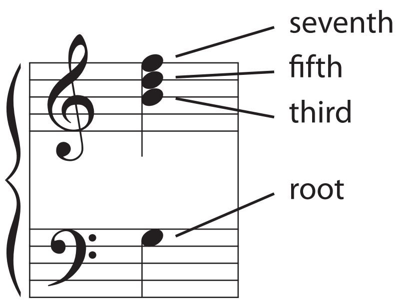


After that you follow all of your tendency tones. *Ti* goes to *do*, *fa* goes to *mi*, and that extra *sol* in the upper voices holds over as your common tone. This will make a normal-looking I chord.

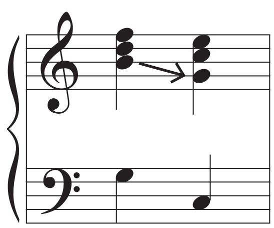
There are a few ways to rotate around your upper tones. I like the formation I've been using above because it fits very comfortably under the hand, but there are two other possible options as well.



Strategy #2 - Frustrated leading tone in an inner voice



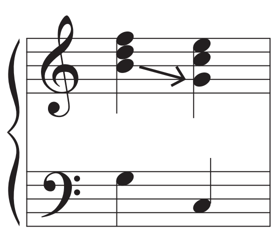
Here we will use all four tones of the V^7. The root is in the bass, everything else is on top, and you should make sure the leading tone (i.e. the third of the V^7) is somewhere in an inner voice.

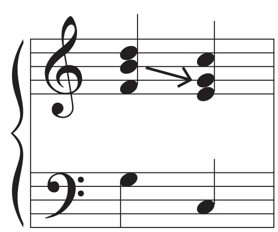


The leading tone ducks down to *sol*. Everything else slides into place.

Because *ti* is in an inner voice, it is not considered "frustrated." (You might say it is frustrated but nobody notices, so life goes on. It may have to do a little therapy later to deal with any residual disappointment.)

Because the third of the chord cannot be on top, there are only two rotations of this voice-leading strategy we can use.

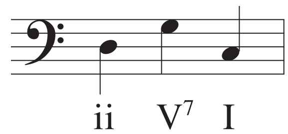




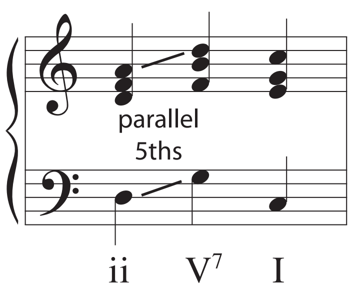
Getting in to your V7

It's pretty hard to get into trouble as you enter into your V7 as long as you make smooth connections and remember your step zone. Let's consider three likely progressions that will contain V7.

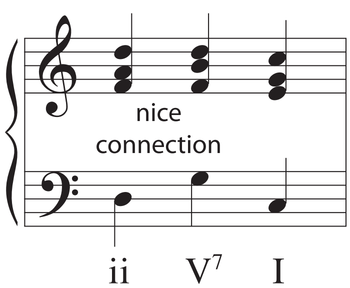
I - V^7 - I and ii - V^7 - I



With either of these, the bass is jumping into the V7 by fourth or fifth, so we can use modified versions of the common-tone or next-closest techniques. Remember that the point of these techniques was always to avoid jumping too much in your upper voices.

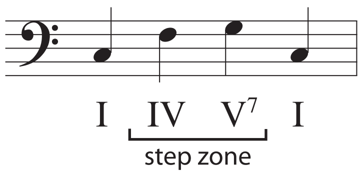


If you jump by 4th or 5th in your upper parts you will probably hook up with the bass in parallel fifths or octaves, or you may make fifths or octaves by contrary motion.

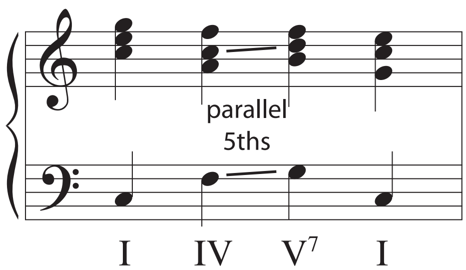


Here we do it correctly by slightly adapting our common-tone technique. This progression makes a very smooth connection into the V^7 by keeping *two* common tones.

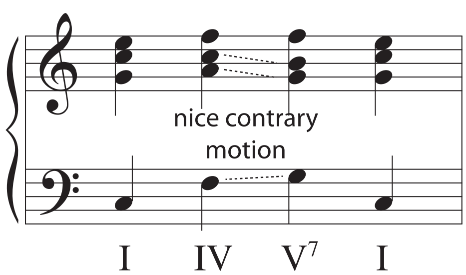
I - IV - V^7 - I



Here there is a step zone from IV to V, so you want to make most of the upper voices go in the opposite direction.



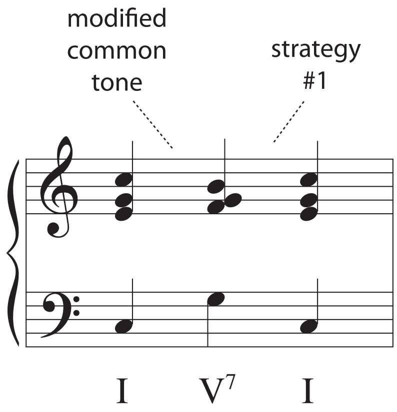
If you don't, you might make bad parallels against the bass.

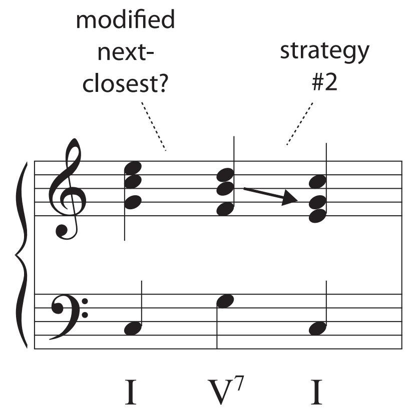


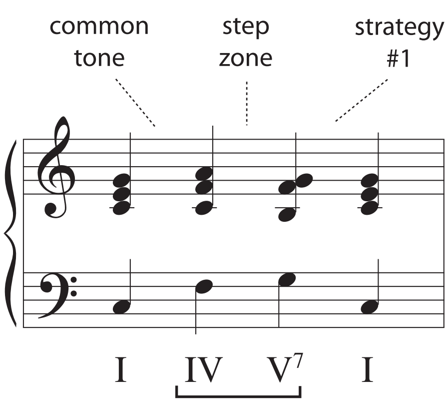
...so we want to pull most (or all) of the parts downward, in contrary motion. Here we hold over the F and move down the other two.

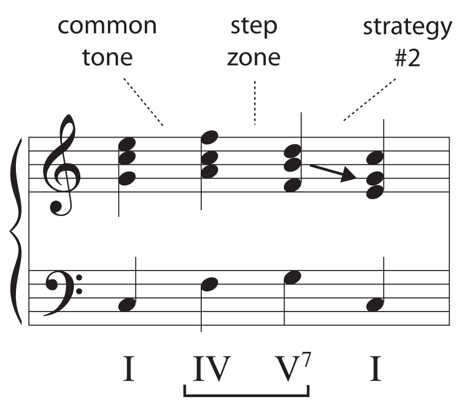
Overall, if you have a good feel for our older techniques and you learn our two new strategies for building and resolving V^7 it should be easy to write these more interesting progressions and avoid problems.

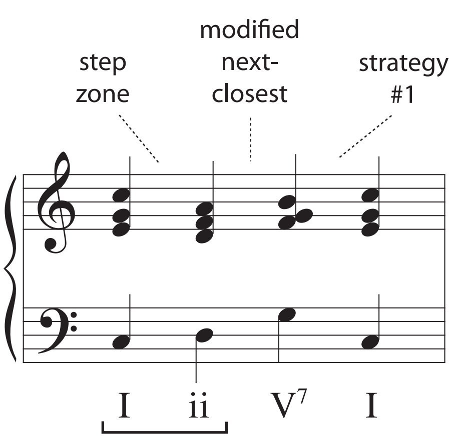
Let’s conclude with annotated examples of various possible progressions that include V^7. This is not an exhaustive listing by any means.

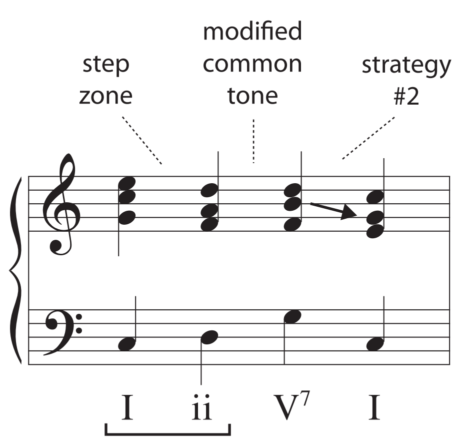






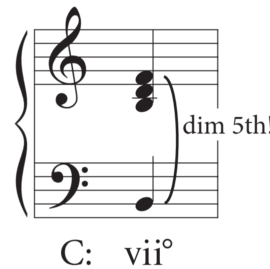




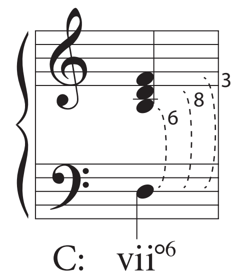


Chapter 3: Playing with vii$^{\circ 7}$

Up until now we've completely avoided the vii$^{\circ}$ chord, and there's a pretty good reason for that. Classical composers didn't like the sound of the vii$^{\circ}$ in root position, because they thought the diminished fifth was too harsh against the bass.

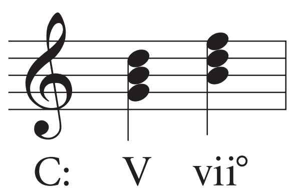


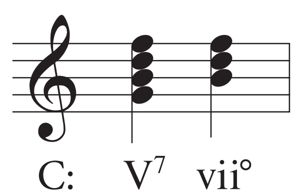
Interestingly, they didn't have a problem with vii$^{\circ 6}$, because putting the chord in first inversion makes all "nice" intervals against the bass.



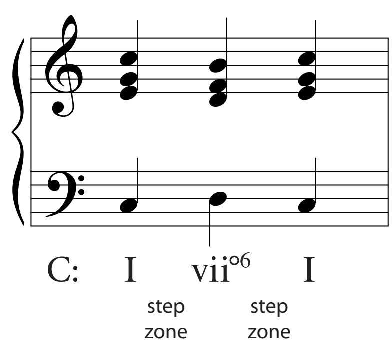
The vii$^{\circ 6}$ triad usually functions as a substitute for the V chord. If you compare the V and vii$^{\circ}$ you can see that they share two notes in common. And, perhaps more convincingly, the vii$^{\circ}$ is equivalent to the upper tones of the V^7 chord.

$$\begin{bmatrix} \text{V} \\ \text{vii}^{\circ} \end{bmatrix} \rightarrow \text{I}$$





If you really wanted to write a I - vii$^{\circ 6}$ - I progression you could pull it off by thinking of your step zones.

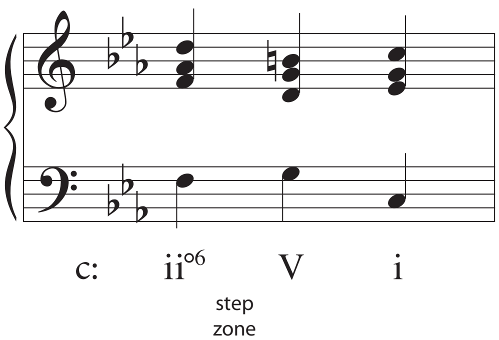


However, we aren't really going to bother with that! Just be mindful that this is something you might see in classical music.

ii$^{\circ 6}$ in minor

Also the same principle holds for ii° in minor. Classical composers were unlikely to use the diminished ii° in root-position, instead preferring ii$^{\circ 6}$. Instead of ii° - V - i they would write ii$^{\circ 6}$ - V - i.

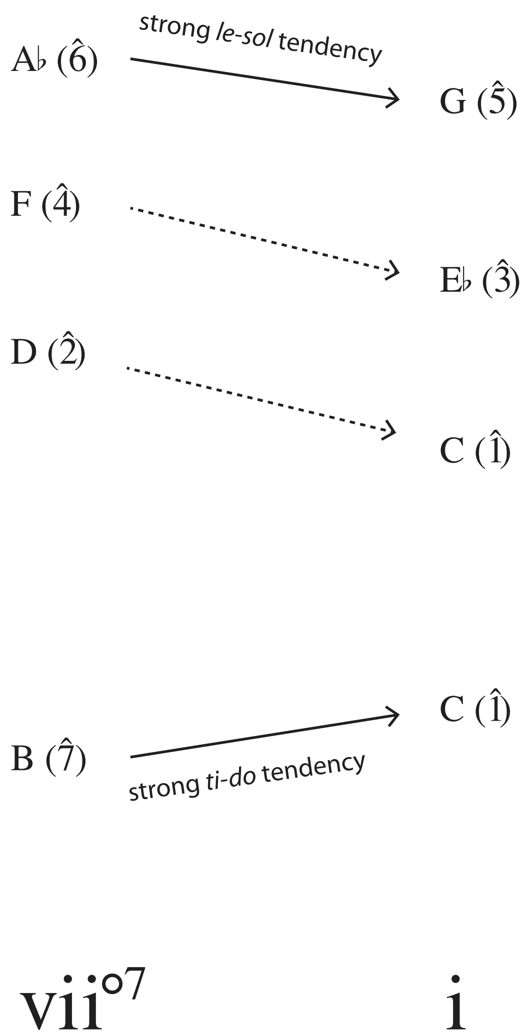
This is actually easy to write if you remember to mind the new “step zone” in the bass line.



But again I don’t think we need to worry about this particular progression.

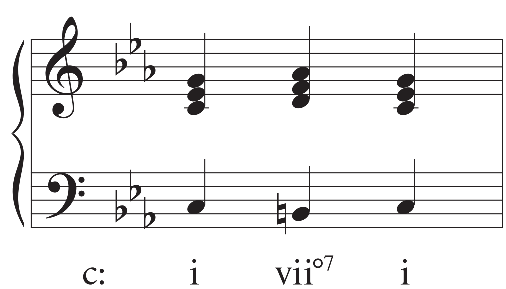
vii°^{7} in minor

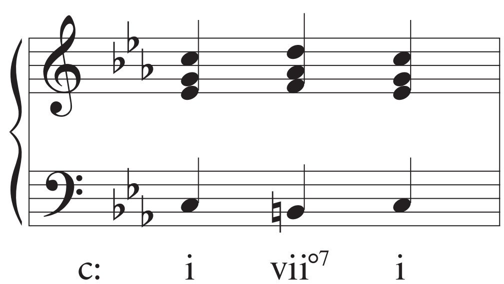
What we WILL practice a little bit is vii°^{7} to i in minor. Like with V^7, the tones in this seventh chord have certain “tendencies” that we need to observe. As long as we follow the tendencies it will be a piece of cake.

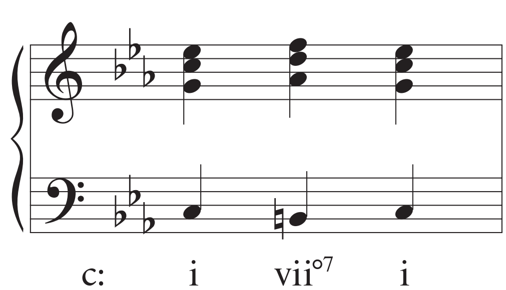


Our root of the vii°^{7} wants to resolve to the tonic, *ti-do.* We’ll take the upper tones and slide them down to the i chord. In particular, the seventh of the vii°^{7} has a very strong tendency to sink downward, *le-sol.*

To keep it simple we’ll do i - vii°^{7} - i, not worrying about other chords that might lead up to vii°^{7}.

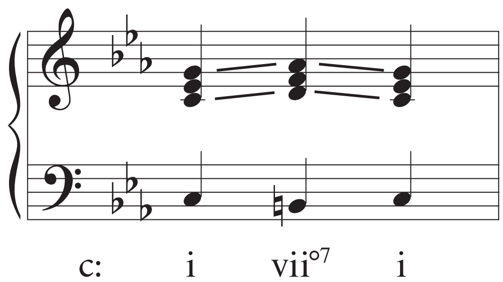






Unequal fifths

Perhaps the most eagle-eyed reader might notice something odd about our basic i - vii°7 - i progression. We don't normally push triad shapes up and down like that. Isn't it making parallel fifths in the soprano and tenor?

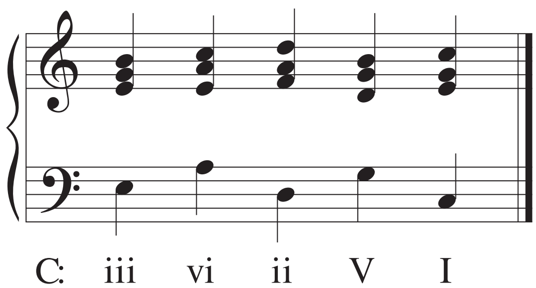


The answer is no, because these are "unequal fifths." C-G is a perfect fifth but D-A♭ is diminished. It's not literally parallel motion so this is considered fine. You are only allowed to get away with this in your upper parts, though.

Chapter 4: Some New Flowchart Details

In our Fundamentals textbook we gradually built up a progressions flowchart as we learned how to make different connections between chords. Let us revisit it now and add a few additional details that pop up frequently in Classical music.

The "spine" of the flowchart is bass notes that fall by fifth. We can start at the ultimate goal, the I chord, think back to V which is obviously a fifth above I, then back further to ii which is a fifth above that. Extending the chain further back ropes in vi and iii. Most people write this kind of progression with a zig-zagging bassline that alternates fourths and fifths.



$$\text{iii} \rightarrow \text{vi} \rightarrow \text{ii} \rightarrow \text{V} \rightarrow \text{I}$$

In addition, we expanded the position leading up to V, including IV as well as ii.

$$\text{iii} \rightarrow \text{vi} \rightarrow \begin{bmatrix} \text{ii} \\ \text{IV} \end{bmatrix} \rightarrow \text{V} \rightarrow \text{I}$$

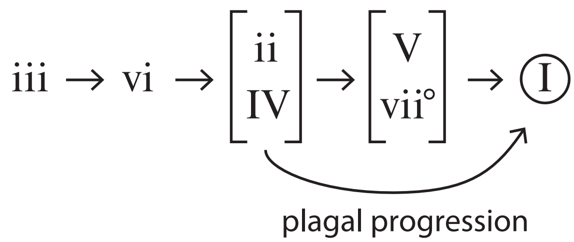
...and in Chapter 3 we noted that vii° is a possible substitute for V.

$$\text{iii} \rightarrow \text{vi} \rightarrow \begin{bmatrix} \text{ii} \\ \text{IV} \end{bmatrix} \rightarrow \begin{bmatrix} \text{V} \\ \text{vii}^{\circ} \end{bmatrix} \rightarrow \text{I}$$

Now it is time to add a few exceptions that frequently occur in Classical-type music.

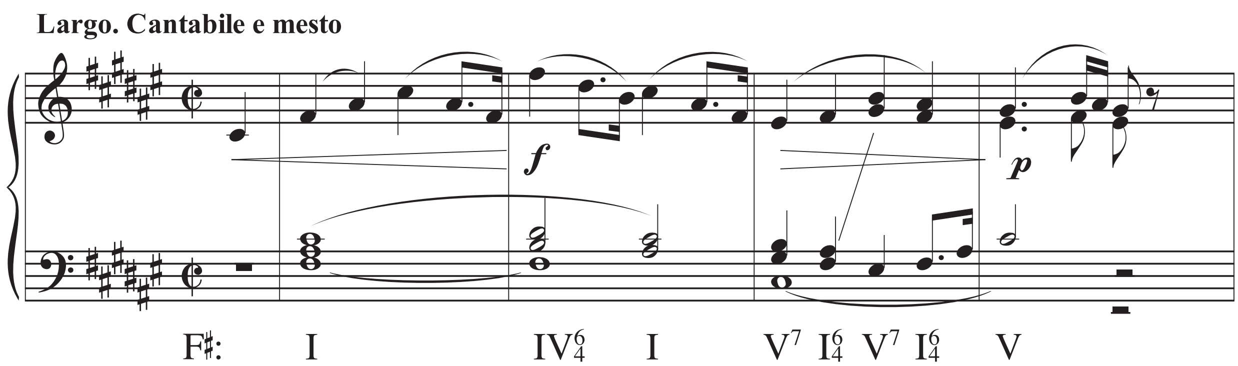
The Plagal Progression

We've talked about this one before. Instead of I-V-I, composers sometimes also like to go I-IV-I. It is the mirror image of the usual progression, going a fifth below tonic instead of above.



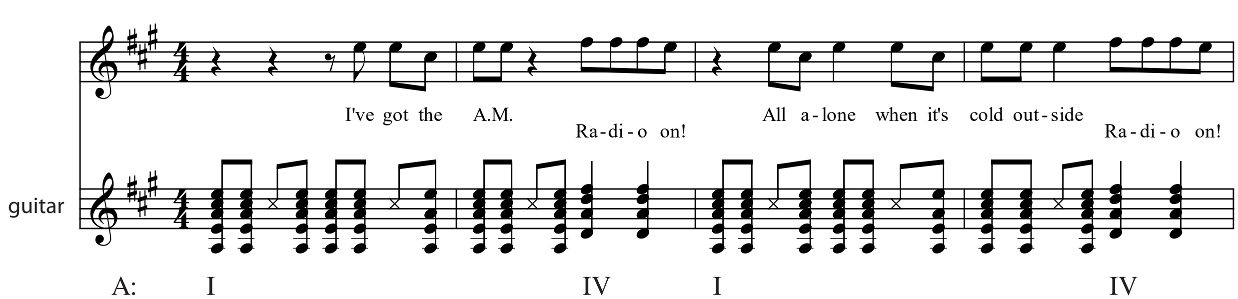
Here's a nice passage from a Haydn string quartet that does it.

Haydn, String Quartet in D major Op. 76 No. 5, second movement mm. 1-4



It's also a very popular move in folk and rock music, which seems to generally favor IV to get a "fresher" sound. One notable tune that jams almost exclusively on the plagal progression is "Roadrunner."

The Modern Lovers, "Roadrunner" [1976] ca. 3:23

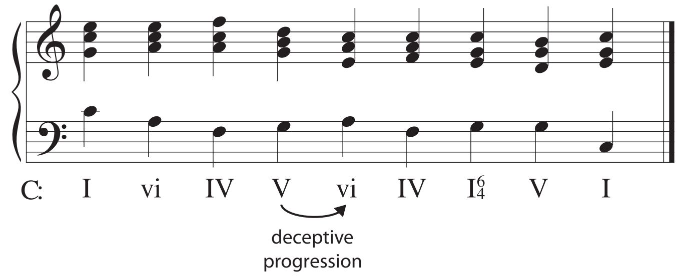


The Deceptive Progression

This is a handy device to extend whatever passage you are writing. You can let your progression proceed to V, but instead of resolving to I as expected you go up to vi. Sometimes this happens at a particularly dramatic moment to create a big "surprise."

deceptive progression

$$\text{iii} \rightarrow \text{vi} \rightarrow \begin{bmatrix}\text{ii}\\ \text{IV}\end{bmatrix} \rightarrow \begin{bmatrix}\text{V}\\ \text{vii}^{\circ}\end{bmatrix} \rightarrow \text{I}$$

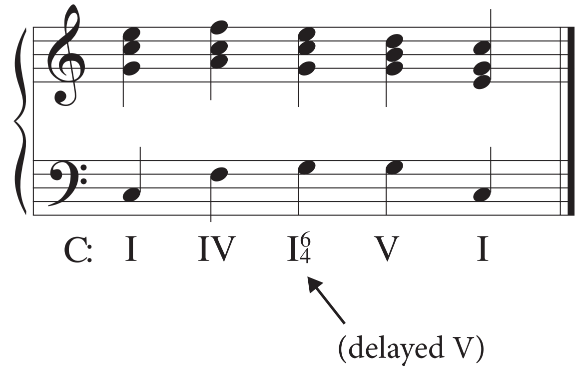


Here is a lovely song from the early 18th century that makes the deceptive move in a minor key.

Parisotti (attrib. Pergolesi), "Se tu m'ami"

The Cadential 6-4

The cadential $^6_4$ is a I chord in second inversion that is inserted before the V. It is a way of delaying the V and making a little more out of it.



We could imagine it as an optional insertion into our flowchart.

cadential 6-4

$$\mathrm{I}^6_4$$

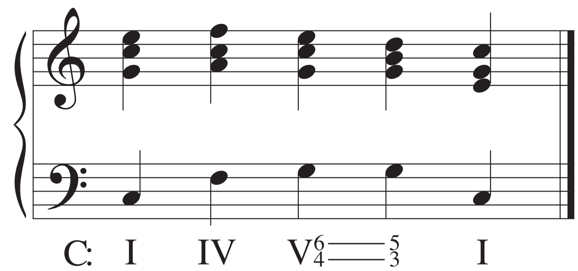
$$\mathrm{iii} \rightarrow \mathrm{vi} \rightarrow \begin{bmatrix}\mathrm{ii} \\ \mathrm{IV}\end{bmatrix} \rightarrow \begin{bmatrix}\mathrm{V} \\ \mathrm{vii}^\circ\end{bmatrix} \rightarrow \mathrm{I}$$

The cadential $^6_4$ is very common in Classical music. Here's an example from a Mozart piano concerto, in which the piano repeats a cute little phrase in dialogue with the orchestra.

Mozart, Piano Concerto No. 9 in E-flat major, K. 271 "Jeunehomme," first movement mm. 1-4

Some people call it V^6_4

There is a certain school of thought that labels the cadential 6-4 as V^6_4, not I^6_4. They would analyze our model progression like so:

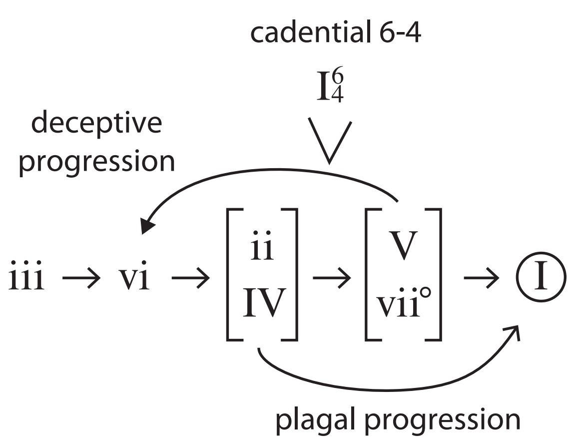


The idea here is that the $^6_4$ is "really" V, and we are just waiting for the tones to fall into place. The little numbers with the lines show that happening. I think this makes sense if you have a lot of experience looking at these little figured-bass numbers, but for the average beginning theory student it is probably nonsensical.

We are not going to do this, and you don't have to worry about it. I'm just warning you that people like this are out there.

Now we've got a full standard diatonic flowchart.

Here it is with all of our new bells and whistles added. The music of Haydn, Mozart, and Beethoven will tend to follow this ordering for a large percentage of the time. I'm calling this the "diatonic" chart because this is what we expect when a composer is staying strictly in the key and only using notes from the scale. One of the last remaining exceptions is *chromatic* modifications to the progression, using notes from outside the key, which we'll consider in Chapter 6.



Chapter 5: Non-Chord Tones

Note: This is mostly just a reprint of Chapter 24 in the Fundamentals textbook. We were only able to briefly consider NCT's in the previous class, so it's time to think about them again.

Most of the music you hear every day is based on chord progressions. A composer chooses a sequence of chords, and the notes in the progression provide a sort of backbone or framework for the music. It is possible to make an entire melody out of selected notes from the chords, like with this familiar folk tune:

However, we usually need more notes to flesh out the music. We add decorations and even interesting distortions to our harmonies in order to make it all sound a little more alive. These extra, added notes are called *non-chord tones*.

In order to illustrate the various kinds of NCTs I'm going to use a few graphic symbols:

● = chord tone (consonant, stable)

○ = non-chord tone (dissonant, subservient to more stable note)

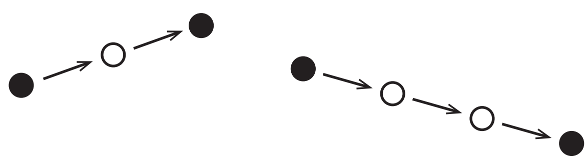
↗ = stepwise connection

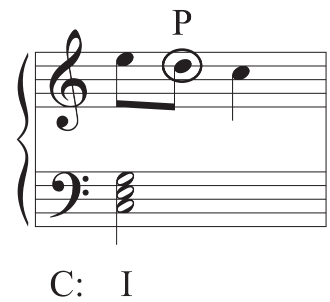
Basic melodic NCTs

First, let's consider some fairly simple ways to elaborate a musical line.

Passing tone

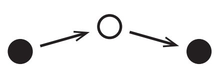
A passing tone comes from a note and continues on to a different note.

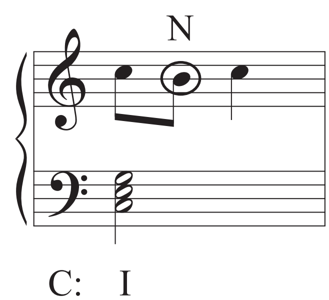




Neighbor tone

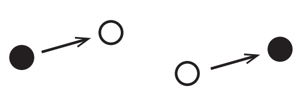
A neighbor tone goes back to the same note it came from.

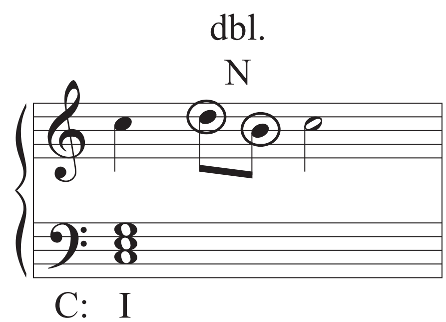




Double neighbor

Instead of going immediately back to the note it came from, a double neighbor figure makes an "above, below, then back" pattern (or the opposite.)

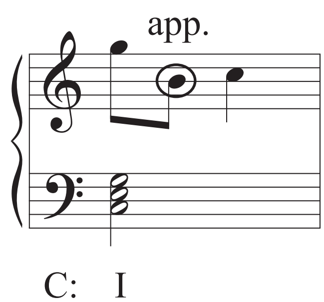




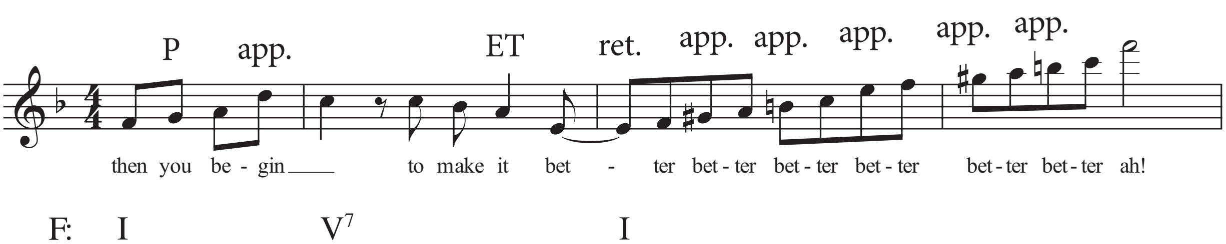
Appoggiatura [from the Italian word "to lean"]

Also known as an "incomplete neighbor." An appoggiatura resolves to a chord tone, but it is approached by leap or just out of the blue.

(not connected) 

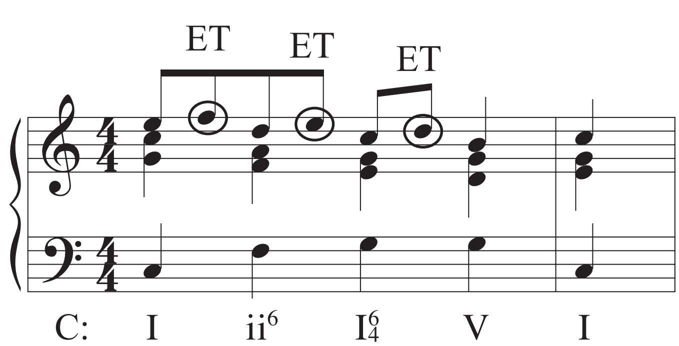


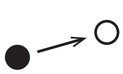
One of my favorite examples of this figure in pop music occurs towards the end of The Beatles' "Hey Jude," as Paul McCartney methodically climbs up the I chord for two octaves, decorating each chord tone with an appoggiatura.



Escape Tone

The escape tone is the only NCT that doesn't resolve to another chord tone - instead it comes *from* a chord tone. It's the opposite of the appoggiatura. The most common use of ET's in classical music is to decorate a scalar ascent or descent, like the example on the right.



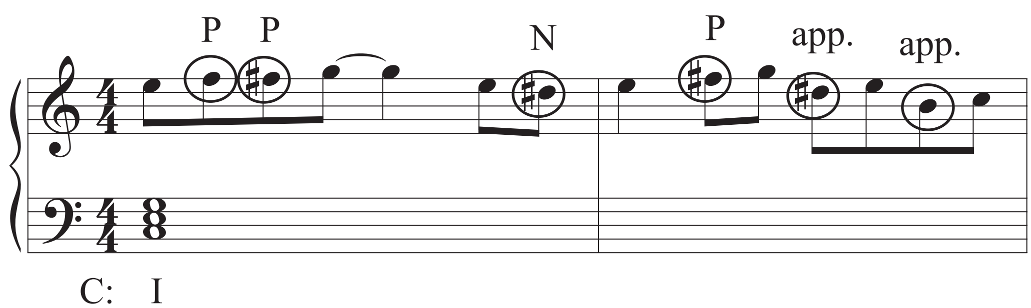
 (not connected)

A few interesting distinctions

Diatonic vs. Chromatic

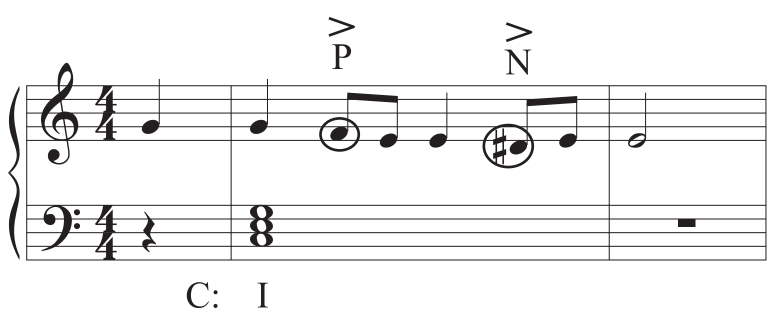
Diatonic NCTs use tones that belong to the scale. Chromatic NCTs use tones that are outside the scale (and require accidentals.)

A passage with mostly chromatic non-chord tones



Accented vs. Unaccented

An unaccented non-chord tone will be relatively "weak" and "unimportant" compared to the notes around it. An accented NCT, however, will stick out - it falls on the beat or it is longer than surrounding notes. You can mark accented P's, N's, or app's with a little accent mark.

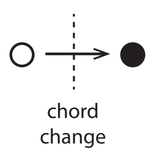


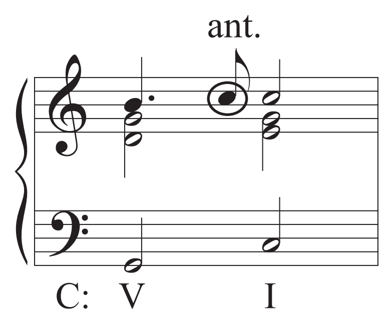
Displacement NCTs - ant., sus., ret., pedal

We also have a few NCTs that involve notes being where they don't belong - they either arrive early or hang on late.

Anticipation

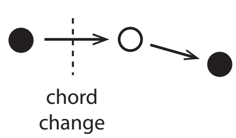
Here a note simply comes in early instead of waiting for the next chord. All of these displacement NCTs can involve either a sustained note (which is held through more than one harmony) or a repeated note. Anticipations are frequently a repeated note.



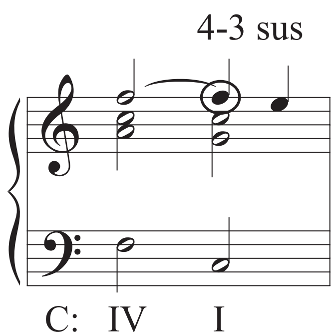


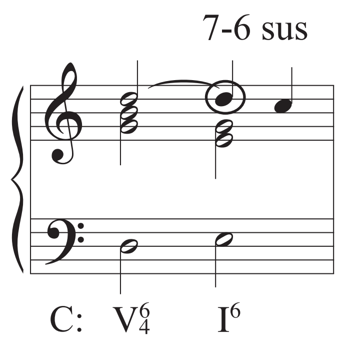
Suspension

Suspensions are perhaps the coolest NCT, but they also involve the most complicated theoretical baggage. Simply put, a suspension hangs over from the previous harmony and then *resolves down by step* into the new harmony.

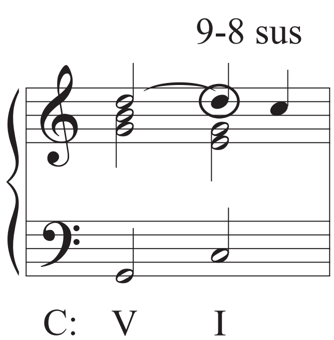


For some reason theorists like to classify suspensions with figured bass numbers. (We aren't really going to worry about it, but you might see technical discussion like this somewhere in the future.) Remember that figured bass numbers are all about the interval above the bass - when you figure out your suspension labels you don't have to think about what the roman numeral is, or the key - just measure the interval above the actual bass note. If you really want to understand this, you could look carefully at these examples and see if you can understand where the numbers come from.

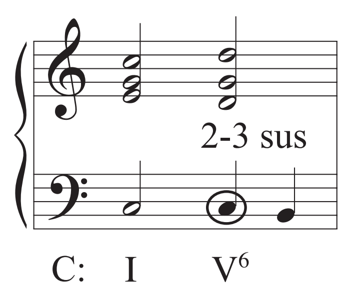




This one is always called "9-8" (even though we might be tempted to say "2-1.")

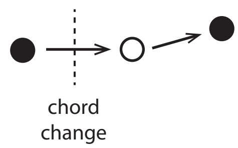


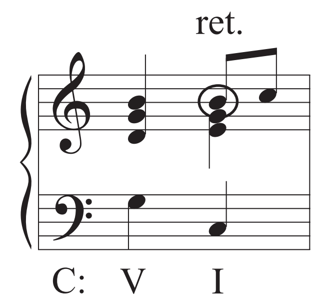
A suspension in the bass is always called "2-3." It refers to the fact that somewhere in the chord there is a tone that the bass is dissonantly rubbing up against - it makes a second against the bass, but once it resolves it makes a consonant 3.



Retardation

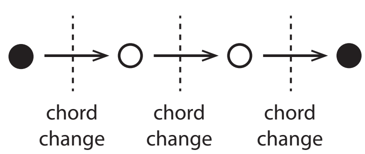
If a held-over note doesn't resolve down, it isn't a suspension. It's a retardation. Retardations are less common than suspensions, and they usually don't get the fancy figured-bass style labels.

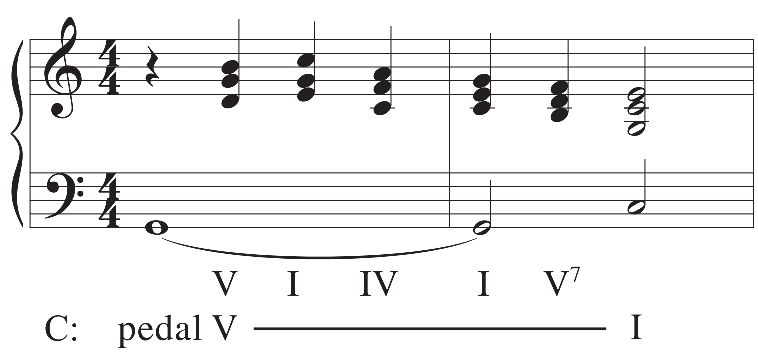




Pedal Tone

A pedal tone is held across a number of harmonies. It is usually in the bass, and usually on scale-degrees $\hat{1}$ or $\hat{5}$. The harmonies on top usually go away from and then back to consonance with the pedal tone. You can analyze it in two layers - on the bottom you can indicate "Pedal V" (or "Pedal I"), and then in an upper layer you can track the harmonies that happen over the bass (with no inversion symbols.)

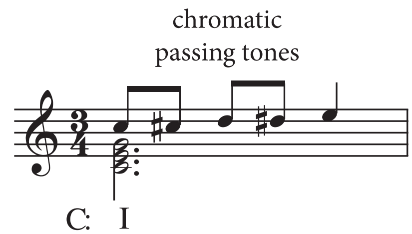




Chapter 6: Chromatic Harmonies

Chromaticism refers to any technique that brings in notes outside the key. (The opposite of chromatic is diatonic, referring to the regular notes that belong to the scale.)

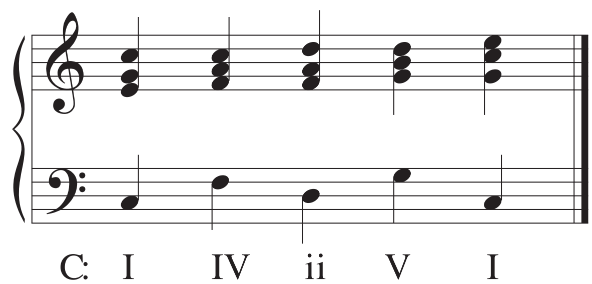
We saw in the previous chapter that it is possible to use chromatic non-chord-tones to decorate your musical figures, like so:



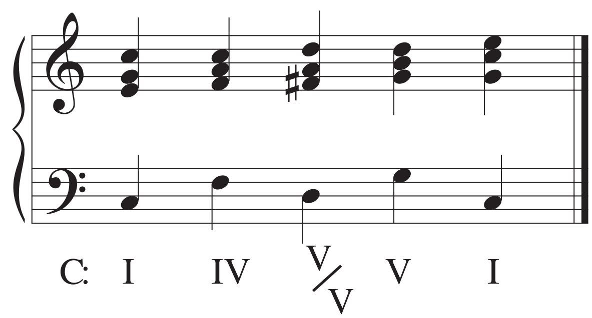
In this chapter we'll consider chromatic harmonies, chords that also pop out of the key in various ways.

Secondary Dominants

Let's imagine that you are writing a fairly basic progression that travels up to V and then home again. We'll make it switch harmonies within the IV-ii box, to spend an extra beat there.

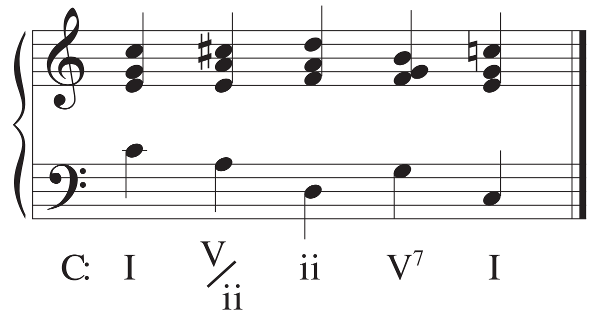


Next, let's snazz it up by making the ii major, like a II. This creates a neat rising motion in our "alto" part, F-F#-G. It you play this progressive you can hear that it sounds kind of wild and aggressive.

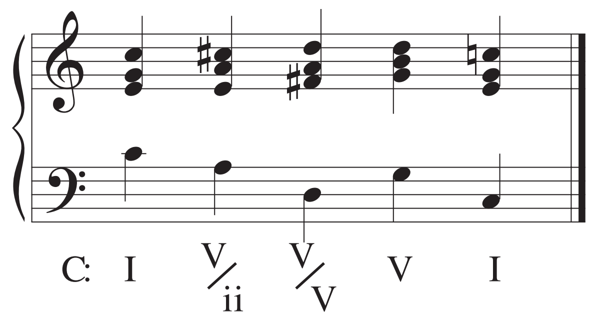


Classical theorists would look at this II and observe that it functions like a V-I in G major. They could call it "V of V" and mark it with a slash notation, "V / V". This is a secondary (or "applied") dominant.

Basically you can change any minor or diminished triad in our fifths-cycle into a major one and it becomes "V of" the next chord in the chain. Here is a progression that includes V of ii.



You can even chain secondary dominants into a sequence - here is a progression that incorporates both V of ii and V of V.



The "V of" language is kind of awkward

Honestly, one thing I don't really like about this convention is that it forces you to constantly refer to the target of the chord rather than the chord itself. Especially when we are chaining multiple dominants together it becomes a little difficult to remember where we are and what is going on.

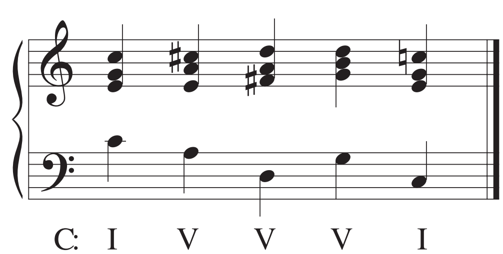
iii → vi → ii → V → I

becomes...

V/vi → V/ii → V/V → V → I

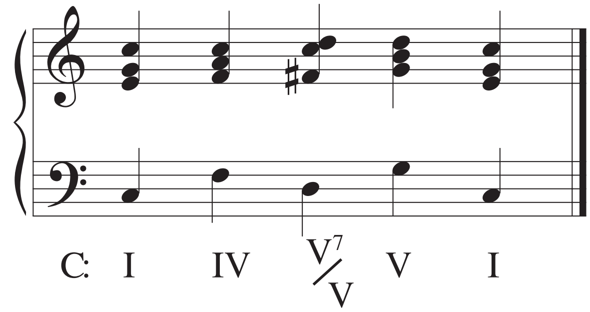
On the other hand, it does describe the effect of these chords pretty well. Secondary dominants tend to disrupt the flow of the tonality a little bit and redirect your attention to a new pitch location, so all of this "V of" language describes what the chords are *doing*.

One slightly less fussy way to notate this in your analyses is to use an arrow that points forward to the target.



Secondary V⁷'s

It's also possible to make these secondary dominants with V7 chords, not just triads. Here's a progression that switches from IV to V^7/V on the way to V. Secondary dominant sevenths are actually very common in Classical-type music.



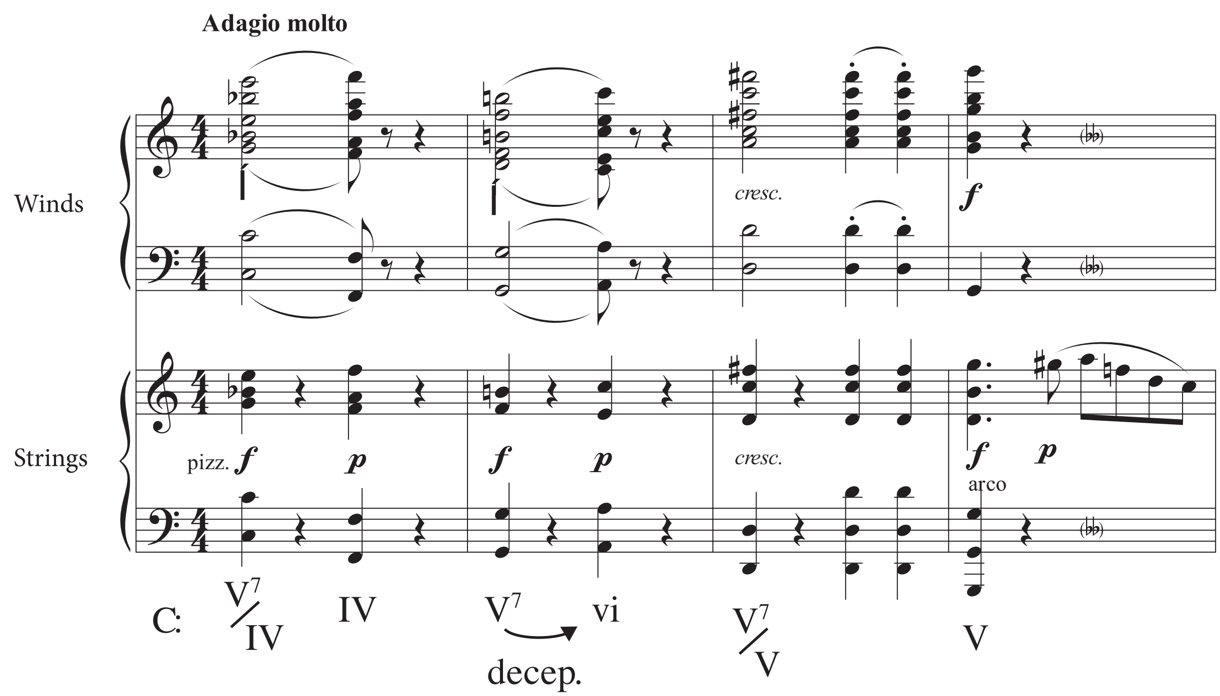
When dominant sevenths chain together in a sequence the tendency tones often work a little differently than usual. Usually we would expect the third of a V7 to resolve up, like a typical leading-tone *ti-do*. However, when two V7's connect the two "juicy" parts of the chord will slide downwards - third connects to seventh and vice-versa. The two tones make a tritone that continues to sink by half-step.

V^7 of IV

V^7 of IV is a particularly interesting case because it is "really" the I chord, which has been turned into a dominant seventh. While the I is usually stable and at rest, V^7 of IV introduces extra dissonance that pushes away from the tonic and jump-starts a new progression.

Beethoven's First Symphony famously starts with a V^7 of IV, followed by a deceptive progression and a cadence on V (which he will eventually have to "undo" as he settles into C major.) This was a bold and intentionally disorienting opening that helped establish his musical personality for his Viennese audience.

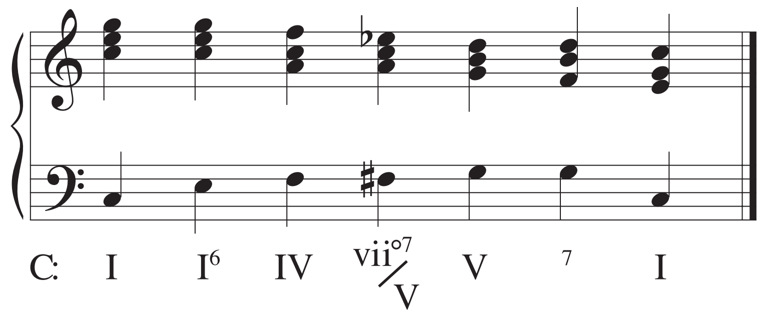
Beethoven, Symphony No. 1 in C major Op. 21, first movement mm. 1-4



Secondary vii°7

It is also possible to insert the vii°7 of any target chord into the progression. This harmony will typically be built on a root that is not in the key, so you can't say that it is "really" a more typical chord that has been altered. Instead it will simply be inserted chromatically, with a root that is a half-step below its target.

Here we'll insert a vii°7/V between IV and V to create some cool chromatic movement.

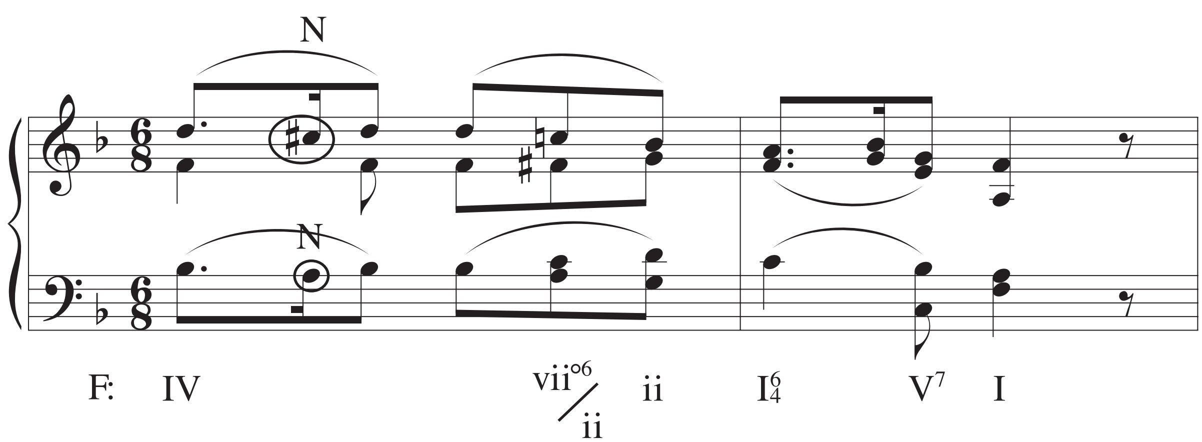


Secondary vii°? Secondary viiø7?

You can also have secondary vii° triads (with no seventh), and even secondary viiø7's, but these are much less common than the fully-diminished vii°7.

Here is a snippet from a Haydn string quartet that sneaks in a plain vii°6/ii on the way up to V.

Haydn, String Op. 20, No. 5 in F minor, third movement, mm. 7-8



Mode Mixture

We will occasionally see chords that pop out of the key but don't seem to be functioning as secondary dominants. One rather vague concept that theorists apply to this phenomenon is "mode mixture," the idea that the composer is "borrowing" harmonies from minor into major or vice-versa. Usually mode mixture injects minor-key chords into a major key.

When we encounter such a borrowed chord there really isn't any special notation to use - you can just apply our usual vocabulary of roman numerals to best describe the chord.

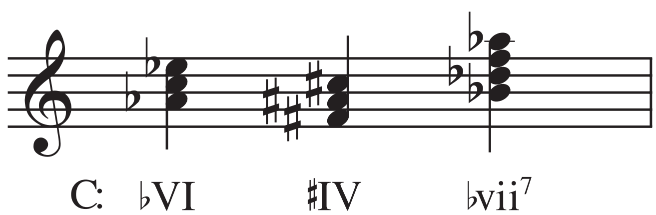
For instance, here is a Schubert song that starts in D minor and shifts to D major at the end. However, even after that happy transition he introduces more reverberations of minor. I would declare the key at this point to be D major and just use minor roman numerals to describe the minor chords that happen.

Schubert, "Ständchen" (from the song cycle Schwanengesang), mm. 27-40

(Schubert, continued)

Using ♭ and ♯ on the roman numerals

If we encounter chords that aren't based on the "proper" scale degrees we can add a flat or sharp to the roman numeral to account for them. Here is a quick sketch of some triads in C major that are "low" or "high" relative to what we would normally expect.



(Of course this labelling technique assumes that there *really* isn't some other explanation for the chord - it's not a secondary dominant and it's not part of a modulation.)

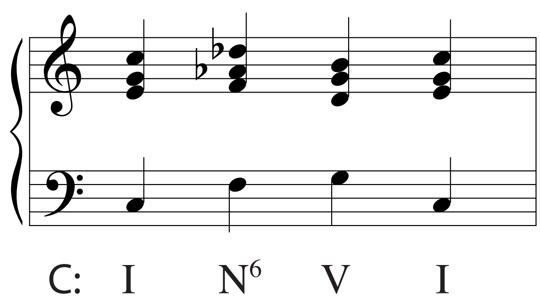
Chapter 7: More Unusual Classical Harmonies

There are a few more special chromatic chords in the typical harmony textbook which we aren't going to worry about in class. However, I will quickly summarize them here! If you are the sort of person who wants to know *everything* about music, this chapter is for you. (Also, if you ever find yourself working on a piece of Classical-type music and see something you don't know how to deal with, you can check back here and see if this chapter helps.)

The Neapolitan 6

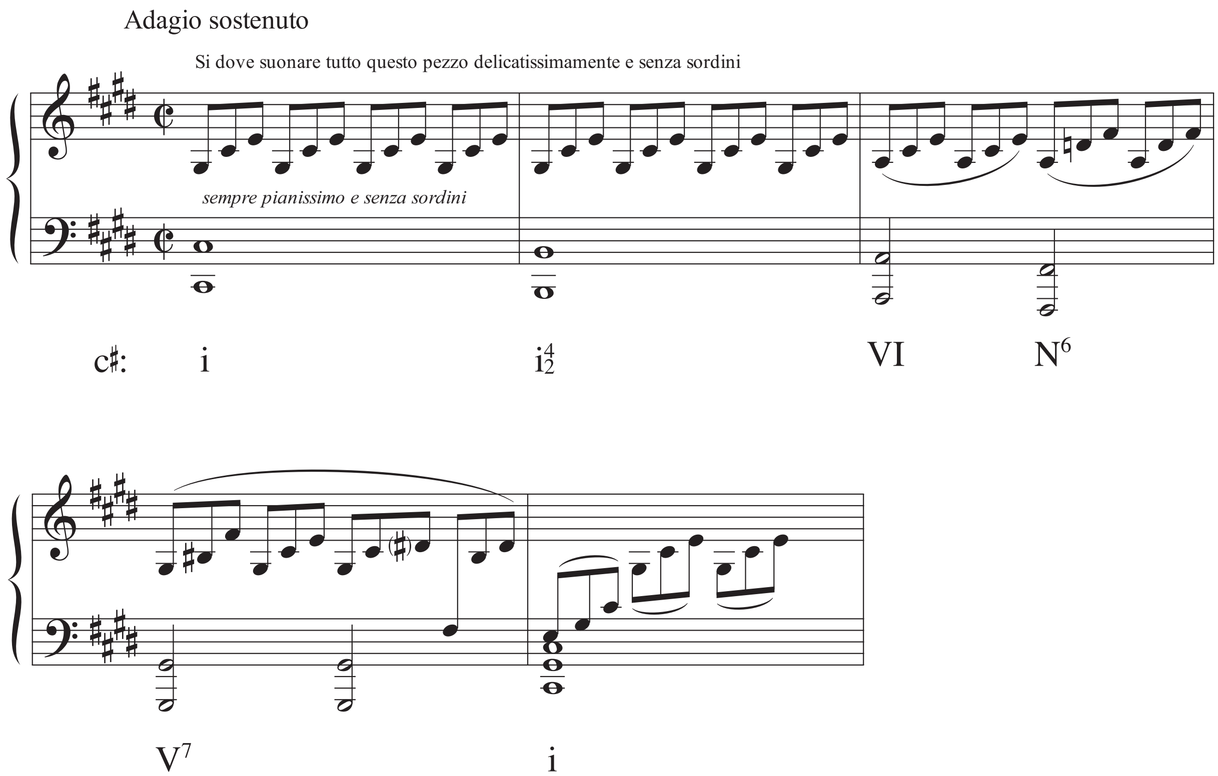
The Neapolitan chord is simply a major triad in first inversion that is based on the flattened scale-degree $\hat{2}$. We *could* label it as $\flat II^6$ instead of the more typical ii^6 or ii^{o6}. Textbooks are enamoured of its special status, however, so they often go with N^6.

Here is a very basic example in C major:



... and the first moments of Beethoven's popular "Moonlight" sonata present a nice example.

Beethoven, Piano Sonata No. 14 in C-sharp minor, Op. 27 No. 2 "Moonlight", first movement mm. 1-5



The Augmented Sixth Chord

The augmented 6th harmony sounds a lot like a dominant 7th chord, but it is spelled in an unusual way that resolves "inside out", usually proceeding to V or I$^6_4$.

There are three variants which have regional names. Let's consider the simplest one first, the "Italian" aug 6th.

The elements of an It+^{6} are...

F♯ → G — a ♯$\hat{4}$, a half-step below the dominant. This wants to resolve up, to $\hat{5}$.

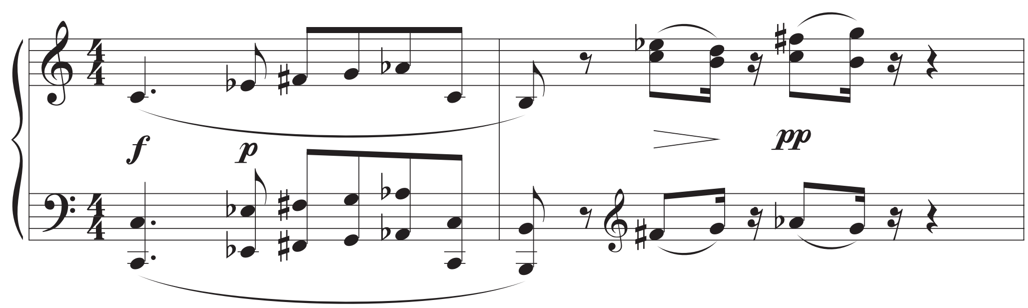
C → D / C → B — one or two tonic notes, $\hat{1}$. These want to resolve up to the fifth of the V chord ($\hat{2}$) or down to the third ($\hat{7}$). If you have two of these in your four-note texture you can split the difference and go both ways, covering the V chord nicely.

A♭ → G — a ♭$\hat{6}$. This also wants to slide down to the dominant.

C: It^{+6} → V

The interval from ♭$\hat{6}$ to ♯$\hat{4}$ is an augmented sixth, a stretched-out span that is the same size as a minor 7th. That's why this harmony sounds like a dominant seventh but does not behave like one.

Mozart does one of these early on in his C Minor Fantasia, K. 475. Here's measures 1-2:



c: i It^{+6}? V^6? vii$^{\circ 7}$ V It^{+6} V

French Augmented Sixth

The French augmented sixth chord adds one new tone to the basic formula of the It^{+6}. Instead two $\hat{1}$'s, this version adds both $\hat{2}$ and $\hat{1}$ for a "whole-toney" sound that is really kind of exotic.

F♯ → G ♯$\hat{4}$ resolves up to $\hat{5}$.

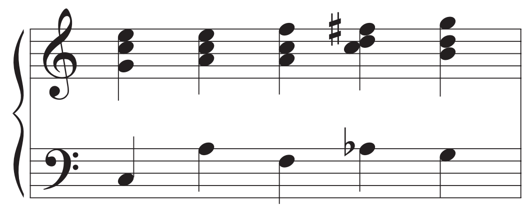
D → D The new $\hat{2}$ holds over as a common tone.

C → B Tonic goes down to B.

A♭ → G ♭$\hat{6}$ resolves down to $\hat{5}$.

C: Fr^{+6} V

Here is a basic keyboard progression, to demonstrate:



C: I vi IV Fr^{+6} V

German Augmented Sixth

The German version adds a minor $\hat{3}$ instead of a $\hat{2}$, which perfectly mimics the sound of a full dominant 7th chord on ♭VI.

F♯ → G

E♭ ⇢ D

Our minor $\hat{3}$ very much "wants" to go to the fifth of the V chord, but doing so will create parallel fiffths with the bass.

C → B

A♭ → G

C: Ger^{+6} V

In this string quartet excerpt Haydn dodges the problem by flipping up his viola line up to the $\hat{5}$, tripling the root of V.

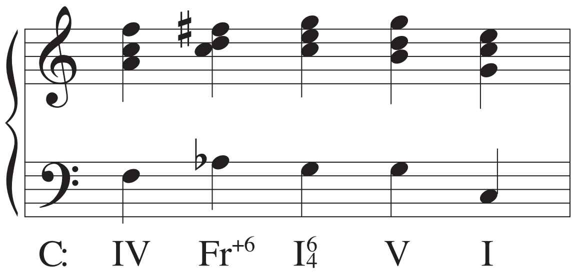
Haydn, String Quartet in E-flat major Op. 76 No. 6, first movement mm. 1-8

E♭: I IV^6 I^6 V$^4_2$ vii^{o6} ii I^6 V$^4_3$ I IV^6 Ger^{+6} V

Augmented Sixth Chord to Cadential 6-4

I've diagrammed all of our augmented sixth chords as though they always go directly to V. However, another common move is to resolve to an intervening cadential $^6_4$.

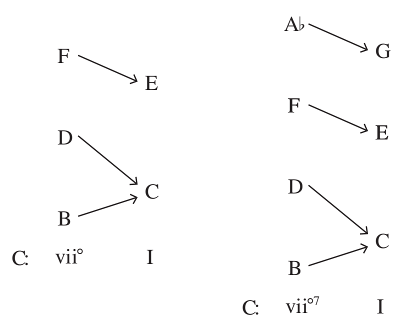
Here is a basic keyboard harmony example:



Other, more-surprising resolutions of the +6 can occur as well, since it can be built on other pitch locations that may lead to other harmonies. The common property of all of these phenomena is the presence of an augmented sixth that expands outwards to some target pitch.

Common-Tone Diminished Chords

So far we've seen lots of diminished triads and seventh chords that resolve inwards, like so:



Occasionally you'll see diminished harmonies that are instead treated like "deflated" major chords. The composer leaves one note on the bottom and reinflates the rest of the triad so that the upper tones slide up.

F♯ → G

D♯ → E

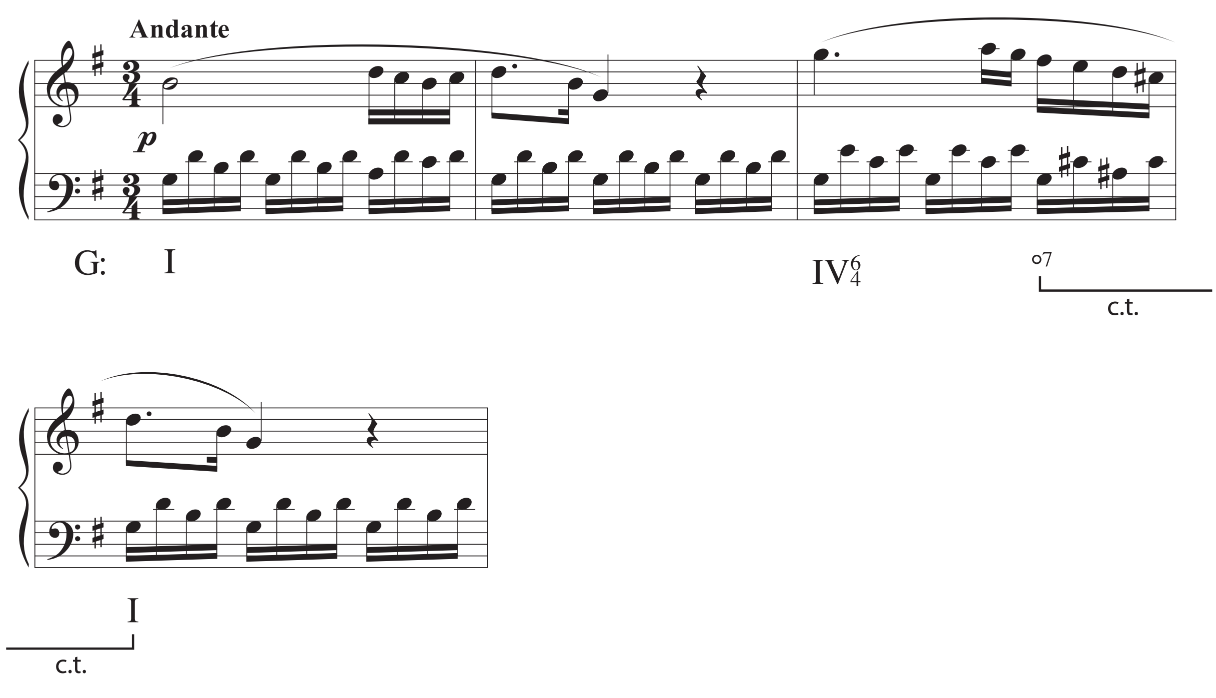
C → C

C: $\circ$ — I
c.t.

The way I learned to mark these is to make the diminished circle, a bracket leading to the target chord, and a "c.t." for common-tone.

Common-tone diminished triads (and sevenths) are frustrating to analyze because they are spelled like a diminished chord with a root right below the third of the target. In the example above, for example, it looks like a D♯ diminished seventh which would be vii$^{\circ 4}_{\ 2}$ of E. However, as you look closer you see that it doesn't resolve the way you'd expect.

Here is Mozart doing it in the second movement of his Piano Sonata in C Major K. 545.



Chapter 8: Phrases and Cadences

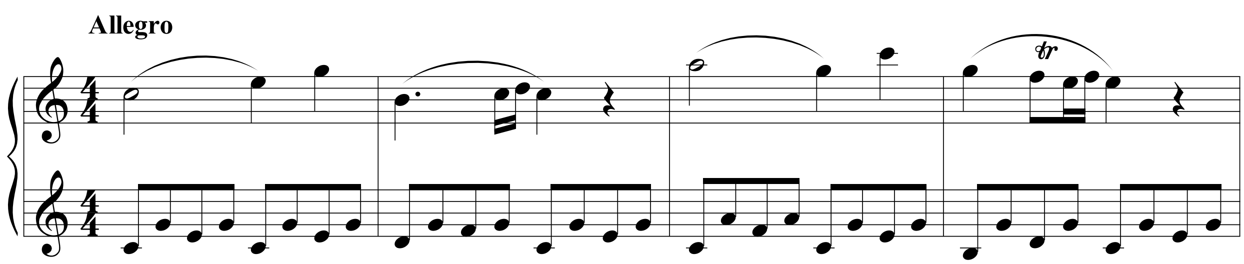
One of the final elements we need before we can really dig in to Classical music is an understanding of phrases and cadences. Taking stock of the way a piece organizes its ideas is an easy and pleasant way to familiarize ourselves with a piece and get the “big picture” of what’s going on.

Phrases

Phrases are just short passages of music with a clear beginning and end. Sometimes these are very obvious, as we hear an idea that starts at one moment, flows onward for a little bit, and then pauses, creating a nice punctuating gap that we can hear with our ears and see in the sheet music.

For instance, this Mozart piano sonata begins with two short phrases. The left-hand part keeps moving, but the melody has clear pauses that we can see and hear.

Mozart, Piano Sonata No. 16 in C major, K. 545, first movement mm. 1-4



I like to diagram phrases with a version of the musical slur shape - the curved ends of the line represent the boundaries of each musical unit and we can indicate the length of the phrase by putting the number of measures underneath.

[Sometimes you can hear phrases even when there are no obvious pauses. I’m eventually going to put the first five or six measures of Bach’s Prelude in C Major from WTC I here as an example.]

Cadences

Cadences are little harmonic and melodic formulas that punctuate the ends of phrases. In Classical music they are very standardized, and we have technical terms to define the different kinds.

Perfect Authentic Cadence (PAC)

The PAC is the ultimate conclusive cadence. It's like the period at the end of a sentence. It will typically go V-I or V^7-I, and maybe it will have a cadential $^6_4$ in there as well. Here is a made-up PAC in our typical four-voice texture.

(I added a little anticipation in there to make it more cadence-like.)

There are certain elements that are required for it to be a perfect authentic cadence.

- The bass line must go $\hat{5}$-$\hat{1}$, making our harmonies in root position. Neither V nor I can be inverted.

- The melodic line must land on $\hat{1}$. It really should have a clear connection from $\hat{7}$ to $\hat{1}$ or $\hat{2}$ to $\hat{1}$.

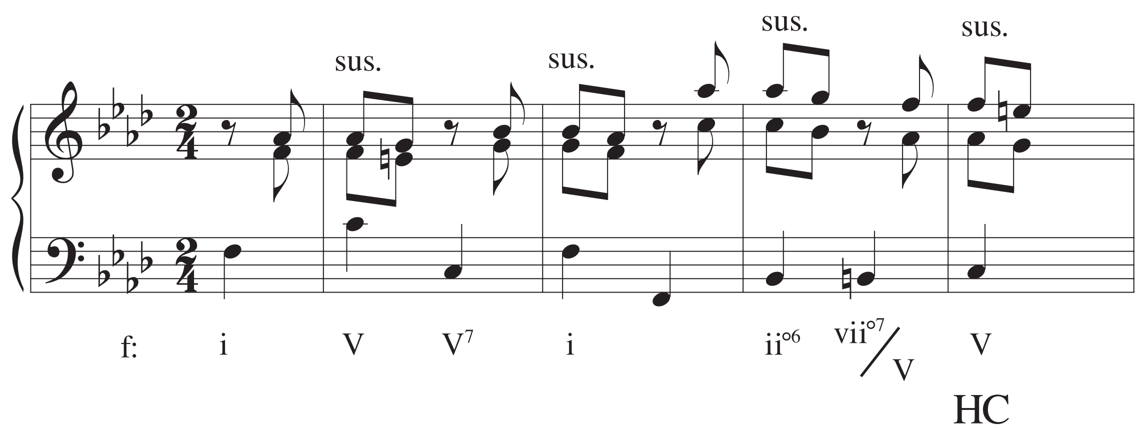
Imperfect Authentic Cadence (IAC)

If either of those two "rules" are undercut, it's an imperfect authentic cadence. An IAC just isn't as conclusive and emphatic as a PAC.

Our Mozart passage begins with two IACs. This makes sense because we are just getting started and our short phrases don't need strong endings.

Half Cadence (HC)

Half cadences simply pause on V. This creates a sense that the music is "up in the air" and still needs to land, demanding more music to finish the thought. It is like a comma instead of a period.



Plagal Cadence (Plag.)

Plagal cadences go IV-I. Like the IAC it is somewhat conclusive but not as emphatic as the PAC.

Deceptive Cadence (DC)

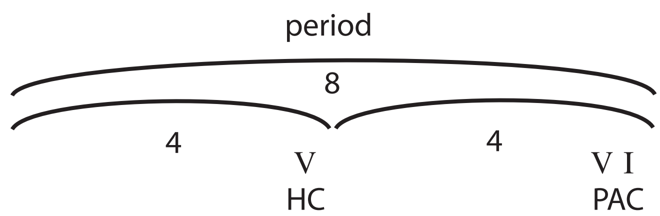
The deceptive cadence sets up a V-I motion but instead goes V-vi! As you can imagine, this is very inconclusive and demands more music.

Anything Else

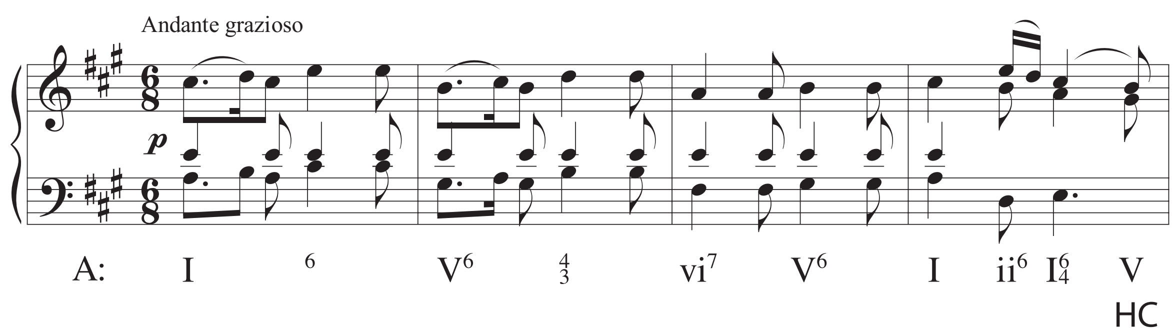
If a phrase ends with anything else you might consider the possibility that there just isn't a cadence there, and the musical thought you are looking at is more like a "sub-phrase." However, you could argue that a pause on anything other than I is some kind of half cadence, and any unusual motion to I (like ii-I) is some kind of IAC.

Periods

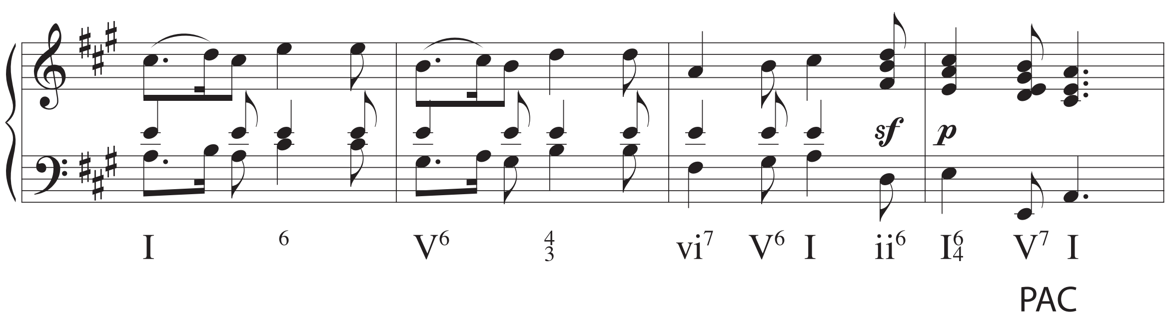
Composers will often write a phrase with a "weaker" cadence followed by one with a stronger cadence, which causes the two passages to hook up and form a larger unit. This is called a *period*. Perhaps the most common design for a period is a half cadence followed by a PAC.



The first movement from Mozart's Piano Sonata in A Major, K. 331 (nicknamed "Alla turca") presents a good example of this kind of phrasing. The first four measures pause on a half cadence:



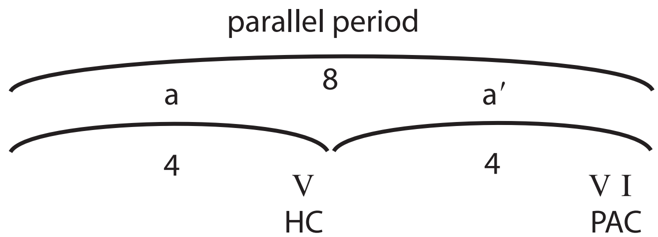
...and then the second four (mm. 5-8) conclude on a PAC.



Parallel Period

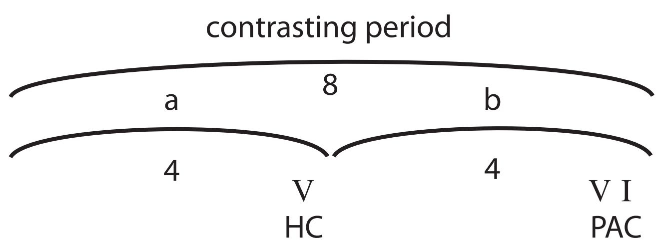
The Mozart K. 331 example from the previous page has two phrases that repeat the same idea, with the second part altered at the end to make a more conclusive cadence. This is a *parallel period*.

We can label the two phrases with a little letter a, to note that the material is the same. The second phrase gets an a′ (or "a prime") to indicate that it has been modified with a new ending.

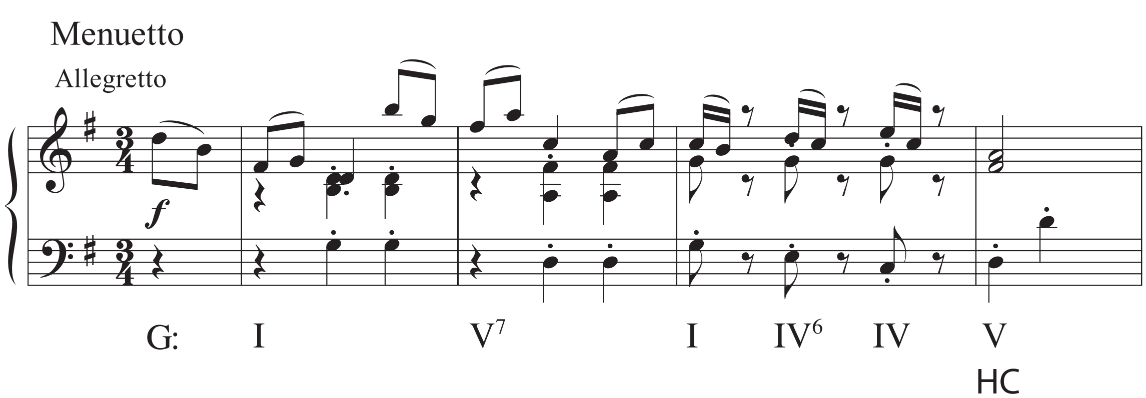


Contrasting Period

The opposite possibility is a *contrasting period*, in which our phrase with the weak cadence is simply followed by something new with a strong cadence. We could label our phrases a and b.



The second movement minuet from Haydn's String Quartet in G major Op. 64 No. 5 presents a nice contrasting period right off the bat. The first four measures present short, punchy figures that lead up to a half cadence:



The next four measures answer with longer, more *legato* figures that lead into a PAC.

Question-and-Answer Phrasing

This kind of phrasing is often called a "question-and-answer" relationship, regardless of whether our answer is a parallel a′ or a contrasting b. In more formal academic writing the first phrase is the *antecedent* and the following phrase is the *consequent*.

Chapter 9: Modulation

Modulation refers to the practice of changing keys after a piece has begun. This phenomenon is central to most Classical-type music, because composers apparently felt that moving from one key to another makes music more interesting. So a typical Classical piece starts in one place (the original key), travels to at least one new key, and eventually returns back to where it started. In certain fairly large-scale forms (like a sonata-form movement) the interplay of keys creates a sort of story or argument.

How do we know when a piece is modulating?

- Sometimes pieces intentionally create the perception that we are "picking up and moving to somewhere else." The musical language may sound very dynamic and unstable.

- In skimming the score (perhaps while listening to a recording) we may notice an arrival on a cadence in a key other than the tonic.

- The underlying scale of the piece may change noticeably, as the composer consistently introduces new flats or sharps that spell a new scale that does not correspond to the key signature.

- As we look at the harmonies and think about a roman numeral analysis, we may notice chord progressions that do not seem consistent with our usual flowchart. Perhaps they make more sense in a new key.

Dealing with modulation in a large-scale view

If you are taking a birds-eye view of your piece (i.e. looking for big ideas and not labeling every chord), you might want to analyze modulation by simply placing a new key indicator roughly where you think the tonality changes and labeling whatever you need to label (like cadences) after that.

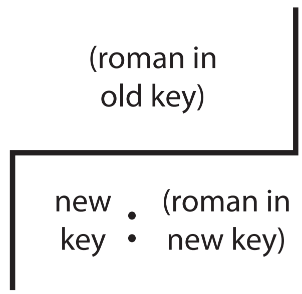
For instance, for homework you may have already looked at J. S. Bach's Minuet in G (from the *Notebook for Anna Magdalena Bach*.) We start in G major, but as we get into the second half of the piece we see a cadence that looks like a big V in D major. We could drop the D: key indicator and mark our half cadence at that point, and maybe that's all we need.

Bach Minuet in G with only the key change and cadence labeled

The Pivot Chord

However, if you are really trying to understand the note-by-note fabric of a piece, you want to know about a traditional analytic concept called the *pivot chord*.

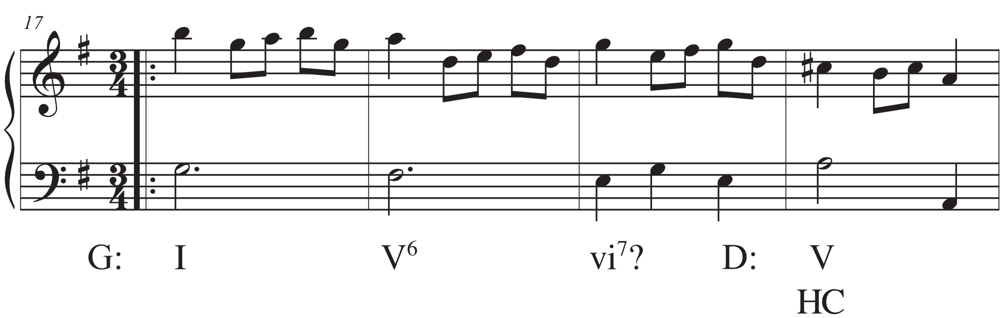
The idea of the pivot chord is that it has a valid flowchart function in both the old key and the new key. To mark your pivot, you want to find the last roman numeral that makes sense in the old key. Draw a sort of zig-zag shape that terminates the old key and begins the new one like so:



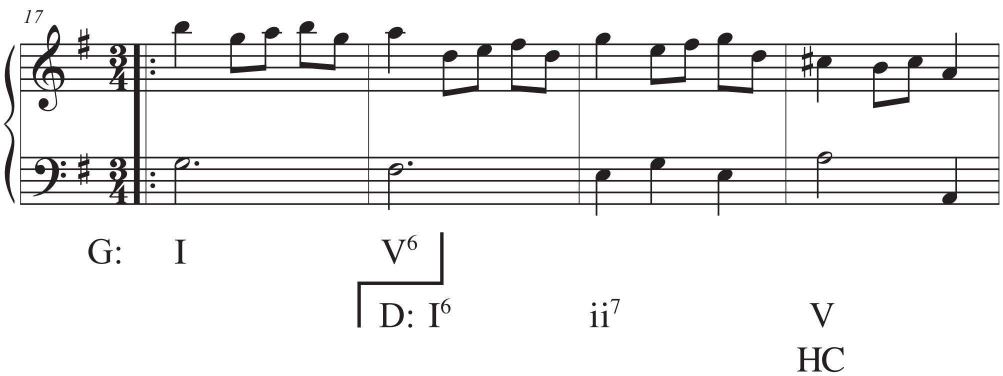
Let's try to do a more detailed analysis of our Bach Minuet modulation, complete with pivot. First, we'll go as far as we can in G major before we are forced to understand things in D major.

Measure 17 is I in G, so that's pretty simple. Measure 18 moves to V^6, which also makes sense in G, and then measure 19 goes on to vi^7, which is maybe unusual but certainly not unheard of. But by measure 20 we are definitely in D major, with a big V that simply doesn't belong to G.

Analyzing the Minuet modulation mostly in G major



I'm going to be very conservative and decide that the vi^7 "doesn't make sense" in G. I'll make a pivot in m. 18 instead, with chord that makes perfect sense in both keys. (See next page.)



So we've decided that the harmony in m. 18 is V^6 in G *and* I^6 in D. Then the chords that follow all make sense in D, with a very textbooky motion from I^6 to ii^7 to V.

If you wanted to put your pivot a measure later and analyze the e chord as both vi^7 and ii^7 I suppose that would also be fine.

The pivot doesn't sound that special

The most counterintuitive thing about the pivot chord is that it probably doesn't sound special - it doesn't "pop out" of the music and signal that something new is happening. Instead, it is the *invisible seam* that connects the two keys as one flows into the other. It is really the NEXT chord (the first one that is definitely in the new key) that is likely to pop out and sound different, but the pivot ensures that the next chord doesn't sound random or arbitrary.

Classical composers had limited pathways between keys

Because Classical composers usually wanted a "smooth" and easy-to-understand transition between keys, they usually limited their tonal paths to related keys that corresponded to chords in the original key. For a major key you can make a little table that includes I, IV and V, as well as the "relative minors" of those (vi, ii and iii.) These are the closely-related keys that would be easy to travel to with a nice pivot chord. (Another way to think of this is that these scales are no more than one flat or one sharp away from the home key on the circle of fifths.)

The closely-related keys for C major

I	C major	vi	A minor
IV	F major	ii	D minor
V	G major	iii	E minor

most likely
first destination

In major keys the most likely destination is the key of the dominant. Sometimes a piece with travel to the V key first and then to more exotic ones later.

Minor keys work basically the same way. We can consider the i, iv and (minor!) v along with their relative majors. Here the most common first destination is the relative major to tonic (i.e. III).

The closely-related keys for C minor

most likely
first destination

i	C minor	III	E♭ major
iv	F minor	VI	A♭ major
v	G minor	VII	B♭ major

Sometimes there is no pivot chord

Sometimes a composer will change keys using some other technique that doesn't involve a smooth pivot. In these situations you can simply drop a new key indicator into your analysis and start analyzing chords with the new set of romans.

Chapter 10: Rock Harmonies

It may not surprise you to learn that the ideas we've developed about chord progressions in Classical-type music have some relevance for other styles, but they are not a perfect fit. Certain practices that are very common with composers such as Bach, Mozart and Beethoven are rarer in more contemporary genres, and popular music has a tendency to do things that a Classical composer would not do. This chapter aims to expand our vocabulary so that we can describe progressions that you might see in rock music.

In general I'm using the term "rock" as a stand-in for many different kinds of music. These ideas here might also apply to chord progressions in pop, jazz, country, R & B, hip-hop, all sorts of styles. (One detail that is fairly rockish, however, is the focus on mostly plain triads, while "jazzier" styles have more seventh chords and other complexities.) I'm going to stick to music examples where there is an instrument playing simple chords, since analyzing harmonies will get more complicated if a track features complicated riffs or other figurations.

I. Rock tunes that stick to our flowchart

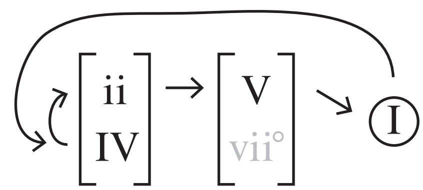
It is not impossible to find popular songs that stick closely to the traditional progressions we've already studied. One example I've cited in class is "12:51" by the Strokes.

```
E                  A
Talk to me now I'm older

     F♯m                      B
Your friend told you 'cause I told her

E                       A
Friday nights have been lonely

F♯m                        B
Change your plans and then phone me
```

Rock tunes usually don't pause on cadences like Classical music does - instead the music is usually more continuous and fluid. Here there is a four-measure cycle that repeats the same four chords over and over again. It sounds like we land on the "home chord" (E major) at the beginning of each cycle. We could analyze all four chords in E major and get a fairly familiar-looking progression of roman numerals:

E: I IV ii V

Obviously this progression starts on the tonic, goes out to a subdominant harmony (IV), flips to a different subdominant (ii), and then proceeds forward to dominant and back to tonic on the next loop.



A deceptive progression

We even see some of our "flowchart exceptions" from Chapter 4 in pop music. You will sometimes hear a tune go from V up to vi in a "deceptive" move that seems to extend the musical momentum.

For instance, "She's Got a Problem" by Fountains of Wayne goes out to vi as it gets towards the end of the verse, in order to set up a final winding path back to I. We are in the key of F major, and I'll quote it starting at 0:49 as it vacillates between IV and V.

```
        B♭              C
She's a danger to herself

        B♭                   C
And I'm worried about her health

            Dm        Am          B♭          C           F
She's got a problem, and she's going to do something dumb.
```

After four bars of lingering on IV and V, it kicks out to vi (D minor), then iii, IV, V and I. The use of iii on the way to IV is a little unconventional, but it does happen occasionally in Classical music as well.

F: IV V IV V vi iii IV V I

deceptive progression

II. Secondary dominants

Rock music also will use certain familiar chromatic chords to push the progression in new directions.

“Sink to the Bottom” by Fountains of Wayne is a very straightforward example of a secondary dominant. Like in our Strokes tune this is another sequence of four chords, E, G♯, C♯ minor, and A.

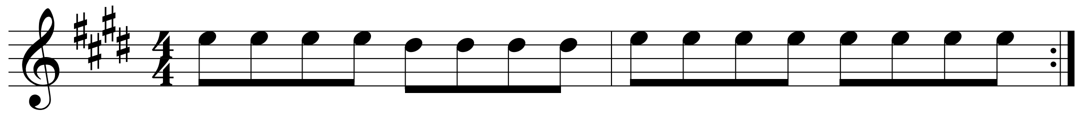
```
         E           G♯              C♯m      A
 I wanna sink to the bottom with you,

E           G♯                C♯m      A
Sink to the bottom with you,

E            G♯            C♯m      A
The ocean is big and blue,          I just wanna

E           G♯                C♯m      A
Sink to the bottom with you.
```

Once again we are in E major. If we just put roman numerals on the chords we could call them I, major III, vi and IV. That III is acting as a secondary dominant to vi (like a V-i in the key of C-sharp minor), so we might as well put that in our analysis.

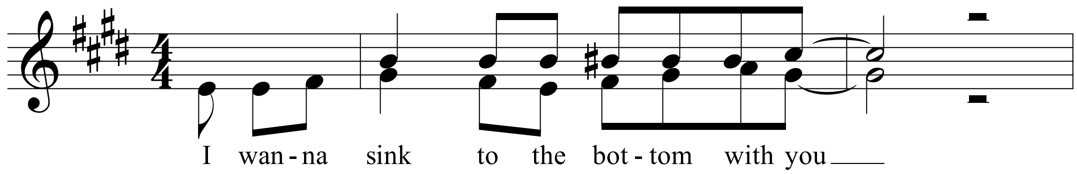
Writing out the progression in our typical keyboard style also shows a few voice-leading paths that show up very explicity in the song.

E: I V/vi vi IV

Our soprano line goes do-ti-do-do, which we hear in the song as a background figure on synth.

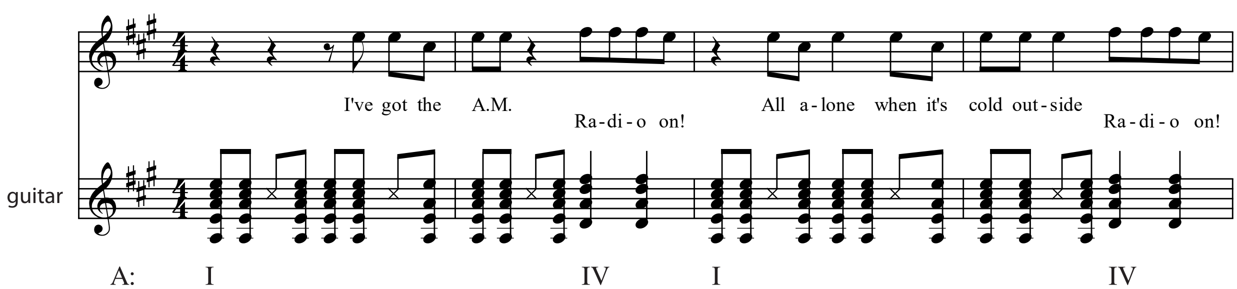


...and the B-B♯-C♯ line in the alto occasionally appears as a harmonizing vocal part.



III. The plagal progression and "plagal stacks"

Popular music seems to like the plagal progression a lot more than Classical music does. My personal theory is that using the IV sounds fresher and less "square" than the more traditional V. In an earlier chapter I cited The Modern Lovers' "Roadrunner" as a tune that relies heavily on the I-IV-I motion.



(The Modern Lovers tune does reserve V-I for the final cadence, however, perhaps recognizing that it is still "stronger" and more conclusive than IV-I.)

Rock music will also happily go V-IV-I, a progression that happens very rarely in Classical music.

For instance, "Let's Dance," originally recorded by Chris Montez (covered by the Ramones et al) has a climactic V-IV-I motion. In G major, we get...

```
             D                              C
We'll do the twist, the stomp, the mashed potato too,

D                      C
Any old dance that you wanna do

          G
But let's dance
```

(This V-IV-I is of course a common feature in the blues as well.)

Plagal stacks

One sometimes hears a sequence of ♭VII, IV and I in popular music. I like to call this a "plagal stack," as the motion from ♭VII to IV is down a fourth, just like IV to I. We could even argue that ♭VII is functioning as "IV of IV," though this concept of a secondary plagal is perhaps not as versatile as a secondary dominant.

Here is a basic plagal stack written out in C major:

C: ♭VII IV I

Let's look at an example in "With a Little Help from My Friends" by The Beatles. Here's the first verse:

```
E              B/D♯       F♯m
What would you think if I sang out of tune?

          F♯m          B7          E
Would you stand up and walk out on me?

E            B             F♯m
Lend me your ears and I'll sing you a song

         F♯m        B7          E
And I'll try not to sing out of key



          D                A            E
Oh, I get by with a little help from my friends

          D                  A            E
Mm, I get high with a little help from my friends

          A                               E
Mm, gonna try with a little help from my friends
```

Once again we are in E major. The first stanza has chords that backpedal from I to V to ii, then proceed forwards again as ii-V-I. When we get to the hook of the song, we see the plagal stack - ♭VII (D major), IV (A major) and I.

Perhaps the most spectacular plagal stack in all of rock n' roll occurs in Jimi Hendrix's "Hey Joe." We are driving towards E major, but we pass through a series of chords that don't seem to belong to the key.

```
C               G
     Hey Joe...

D               A
                   Where you going with that

E
gun in your hand?
```

This is a series of four plagal chords in a row, a chain that runs from ♭VI all the way to tonic.

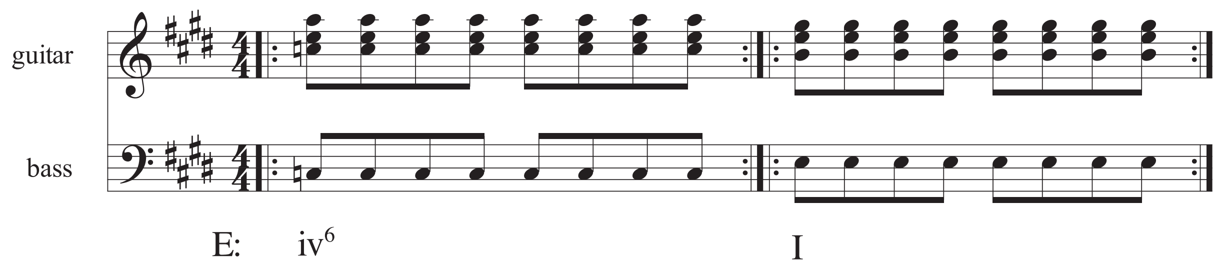
In considering progressions with extensive plagal action it is tempting to forge a second axis for our progressions flowchart. The horizontal axis might be our traditional one that proceeds largely by falling fifths, and a new vertical axis could move by falling fourths.

$$\begin{array}{c} \flat\mathrm{VI} \\ \downarrow \\ \flat\mathrm{III} \\ \downarrow \\ \flat\mathrm{VII} \\ \downarrow \\ \mathrm{IV} \\ \downarrow \end{array}$$

$$\mathrm{iii} \rightarrow \mathrm{vi} \rightarrow \begin{bmatrix} \mathrm{ii} \\ \mathrm{IV} \end{bmatrix} \rightarrow \begin{bmatrix} \mathrm{V} \\ \mathrm{vii}^{\circ} \end{bmatrix} \rightarrow \mathrm{I}$$

IV. Mode mixture

You may recall that mode mixture simply involves flipping between major and minor, usually injecting minor-key chords into a passage that is predominantly major. We see this in the introduction to the Strokes' "12:51", which alternates an E major chord (I) with A minor in first inversion (iv^6). The latter chord is "borrowed" from the parallel minor to create a more interesting beginning to the song. (It will later create a little tonal contrast when the same harmonies come back as the bridge.)

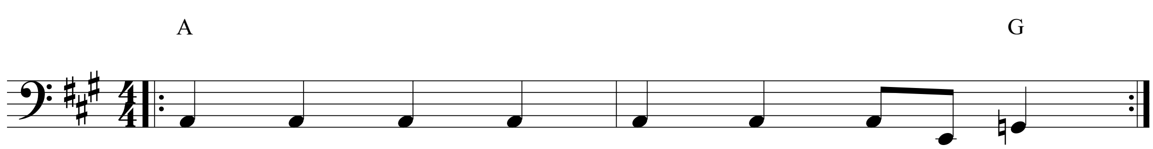


V. Neighbor chords

The Schenkerian theory of tonal music (which is highly influential for theorists of Classical music) argues that there is a sort of continuity between the decorative tones you might see in a melody, the harmonies underneath, and whole key areas. Schenker argued, for instance, that a composer will modulate to a new key in order to create a large-scale passing tone or neighbor-tone that will help take us from the beginning of a piece to the end, just as they might insert an extra eighth-note into the melody to quickly travel from *mi* to *do*.

One phenomenon which kind of forces us to think of harmonies like they are melodic tones is when they move up or down by step, in ways that are otherwise unexplainable. In these situations we might call them passing or neighbor chords.

The main groove of "Psycho Killer" by the Talking Heads has a simple, somewhat hypnotic harmonic scheme that revolves around a neighbor chord. Here is Tina Weymouth's bass line with guitar chords indicated above it:

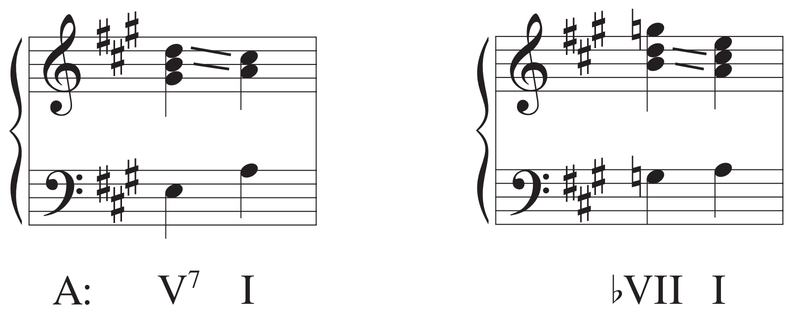


We are basically just "vamping" on A major which serves as our I chord. A more traditional (and some might say corny) approach would be to mix in a little motion to V to support our A and break up the monotony. The Heads choose to dip down to the ♭VII instead in a move that really seems like a lower neighbor to A.

Related topic: Can ♭VII be a substitute for V?

In our Talking Heads example, the G major chord is being used to provide a quick contrast to A. That's usually the V chord (E major)'s job. Could we argue that ♭VII is acting as a new kind of dominant here?

I think we can. Let's compare V^7-I and ♭VII-I in A major.



Here I've written two fairly straightforward progressions, one with a traditional V^7-I and one with a ♭VII-I. I wrote the second one in contrary motion to avoid the massive parallels that would otherwise be present.

Perhaps the signature element in a I-V-I progression is the move to the leading-tone and back, *do-ti-do*. You don't quite get that with I-♭VII-I, instead you get *do-te-do,* so that's only sort of like a dominant. However, you can get other distinctive voice-leading motions from both sides. Both can support *re-do* and both can go *fa-mi*.

So it seems that ♭VII can act like a dominant, and in contexts like our Talking Heads song it seems reasonable to assert that it does. Since this notion is not yet standard in the music theory world it would probably require some explanation if you want to include it in an analysis.

VI. Passing chords

Passing chords are perhaps even more straightforward than neighbor chords. Whenever you see a progression that simply moves by step through multiple chords, it seems reasonable to deem the middle harmonies as passing, there to serve the larger motion by filling it out.

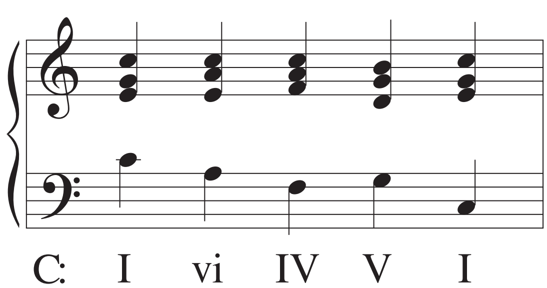
One very familiar progression that does this is i - VII - VI - V in minor keys. You might know this as "Hit the Road Jack," or maybe "Stray Cat Strut" by the Stray Cats.

```
Cm                 B♭        A♭           G
Black-and-orange stray cat sittin' on a fence,

Cm          B♭             A♭           G
Ain't got enough dough to pay the rent.

Cm        B♭        A♭           G
I'm flat broke but I don't care,

  Cm
I strut right by with my tail in the air.
```

This is a motion from the tonic in C minor down to the V, with VII and VI serving as passing chords.

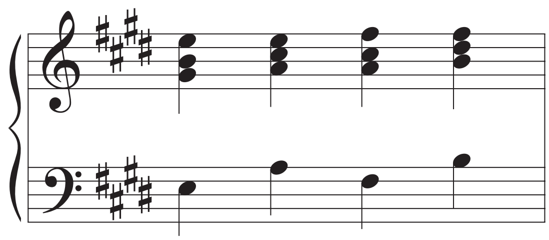
VII. Chromatic third motions

Third motions can be a nice way to slide through a progression, creating a subtle shift from chord to chord. In Theory I we like to practice writing this "Heart and Soul" sequence, which travels down from I to IV before retuning home.



This is easy to do because there are two common tones in each chord connection. That's also why third progressions sound so smooth and gentle.

We also know that some third-related harmonies are good substitutes for each other, like IV and ii. Our progression from The Strokes' "12:51" pivots from one to another on its way to V, like so:



E: I IV ii V

T S S D

I added a layer of analysis here that indicates tonic, subdominant and dominant function. There are some musicians who prefer marking up chords with this T, S, and D system instead of roman numerals.

Chomatic thirds

Chromatic third motions are a little different than these subtle shifts, however. These are progression where the the chords do not fit together perfectly in the key. They will *sort of* have two common tones, but one or both with be altered with a new sharp or flat. The effect is usually much more dramatic and disorienting than a simple diatonic progression by third.

Let us consider a hypothetical move from I to ♭III in C major.

C: I ♭III

chrom 3rd

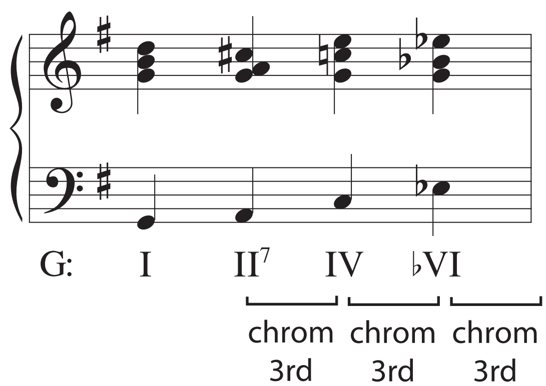
It's pretty clear how these chords should connect. They have one proper common tone, G, and one sort-of common tone that mutates a bit with an added flat (E to E♭). Because the ♭III seems to pop out of the key, this move sounds pretty dramatic and disorienting.

We'll mark these kinds of moves with a bracket and label.

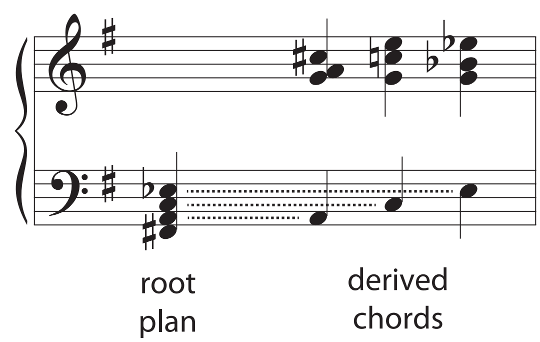
One notable passage in rock with chromatic third motions is the climax of the Talking Heads' "Memories Can't Wait."

```
G              A7          C           E♭
    These memories can't wait!

(repeat 8x)
```

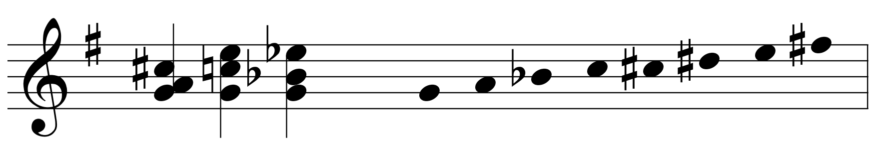
The last two chords are chromatic motions up by minor 3rd, and as we loop back around to G major that forms one last chromatic relation by major third.   Here is a keyboard progression all worked out:



A student from Spring '24 (Nick Moretti) has pointed out that the last three chords in the cycle seem to be following the path of a fully diminshed seventh chord, as if they were part of the same tonal complex:



The chords mesh together to form what music theorists call the *octatonic scale*. (Jazz musicians, on the other hand, sometimes call it the "diminished scale.") It alternates whole and half steps all the way up.



But anyway, this octatonic business is probably neither here nor there. If you listen to this passage you'll hear how chromatic third motions can create a very dramatic effect.

IX. Modulation

Popular music doesn't modulate as much as Classical music does, but sometimes a tune will "go someplace else" from where it starts out. This might occur in a particularly dramatic section (like, say, the bridge of the song) or the tune might simply end in a different key from where it started.

For an example, let's take another look at "Memories Can't Wait." I'll give an abbreviated overview of the whole song with analysis. This composition even has points where we can take a pivot chord, which accounts for the seamless feeling of our journey from C♯ minor to G.

OPENING SECTION (:01)

```
C#m (with some F# in bass)
Do you remember anyone here?
```

c♯: i

```
No you don't remember anything at all...
```

HOOK (:31)

```
A/E            A#/E          E
    There's a party in my mind,
```

VI

E: IV ♯IV I

```
A/E            A#/E           E
     ...and I hope it never stops
```

IV ♯IV I

INTENSIFICATION (:53)

```
B                   C
Other people can go home
V                   ♭VI
```

```
B                  C
Everybody else can split...
V                  ♭VI
                   G: IV
```

POSTLUDE (2:22)

```
G    B                   F    C
Everything is very quiet,
I    III                 ♭VII IV
chrom 3rd
```

```
G            A       E
Everyone has gone to sleep.
I            II      VI
                     chrom 3rd
```

```
G        D         F    C
I'm wide awake, on memories
I        V         ♭VII IV
         chrom 3rd
```

```
G        A7         C      E♭
These memories can't wait!                (repeat 8x)
I        II7        IV     ♭VI
         chrom 3rd  chrom 3rd  chrom 3rd
```

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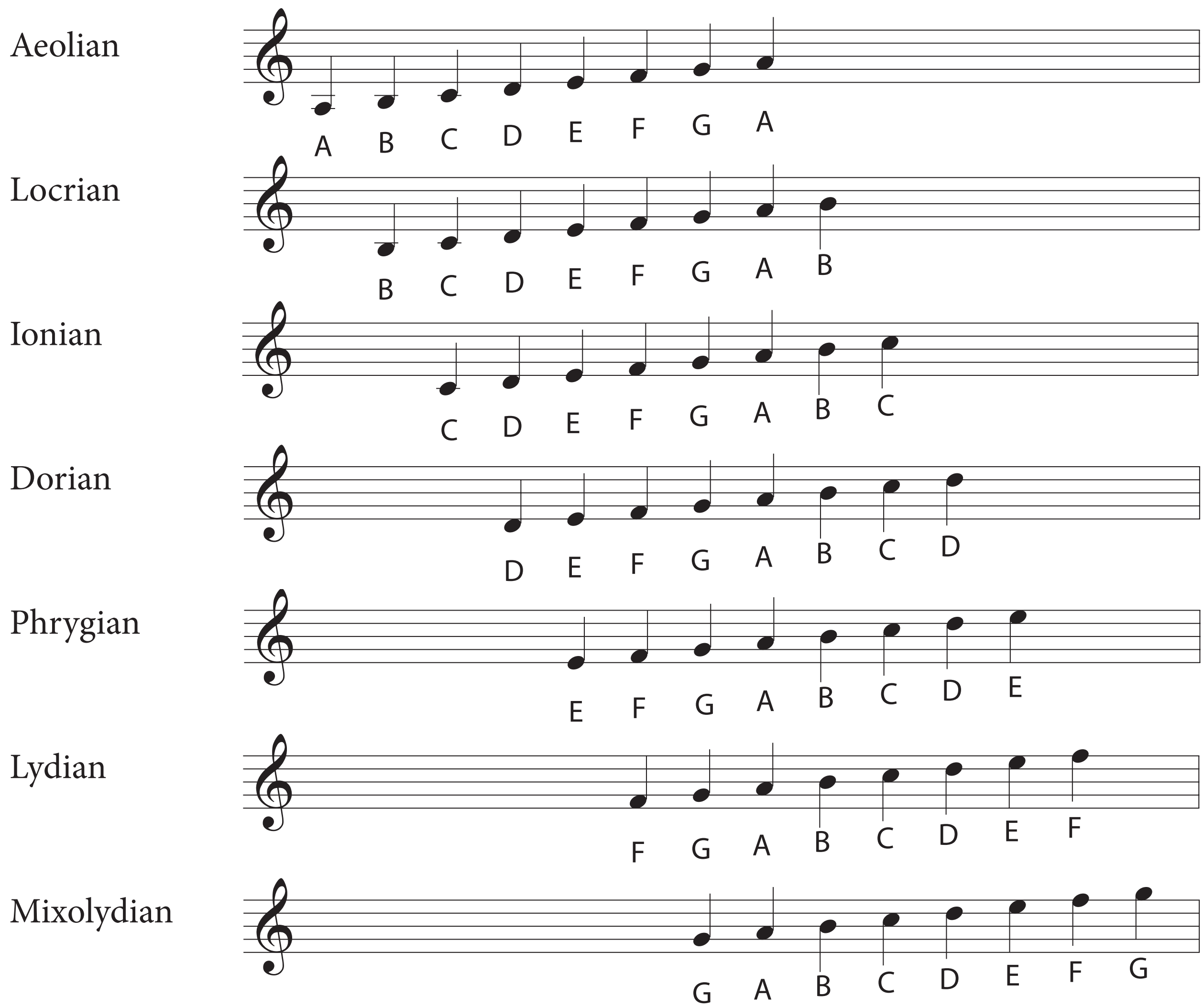
Chapter 13: New Scales and Collections

Modes

European musicians in the Middle Ages lived mostly in a white-note world. They used sharps and flats very sparingly, and didn't have anything like the system of 12 major and minor keys we have today. Instead, they achieved tonal variety through the *church modes*, a system of seven different scales that can be derived from the white notes of the piano. They gave them somewhat fanciful geographical names inherited from the Greeks.

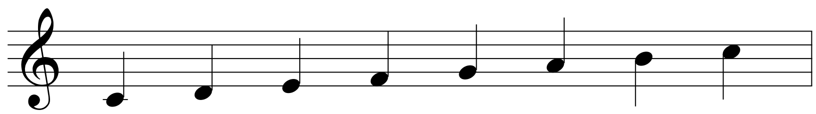
We already know that you can derive the major scale by playing from C to C on the piano, and the "natural" minor is A to A. Imagine more scales generated from every possible segment of the white keys.



Let’s organize these scales another way. We’ll transpose them so that they all start on C. We’ll group all of the major-type scales together (i.e. those with a major $\hat{3}$) and also all of the minor-type scales (with the minor $\hat{3}$).

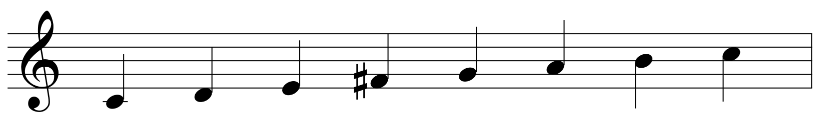
Majorish Modes

C Ionian



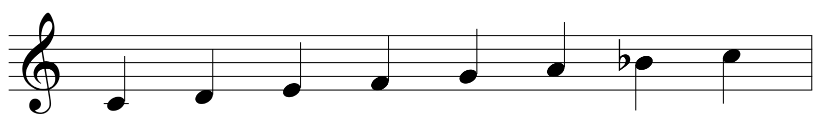
This is the standard major scale.

C Lydian



Major with $\sharp\hat{4}$

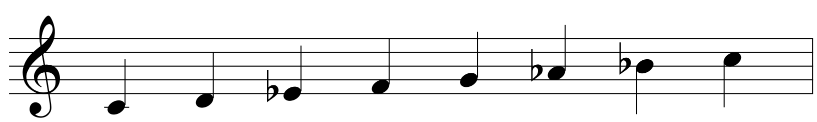
C Mixolydian



Major with $\flat\hat{7}$

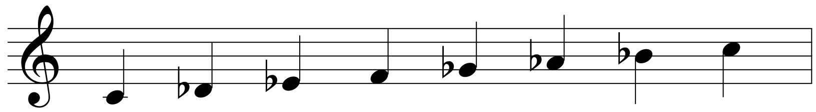
Minorish Modes

C Aeolian



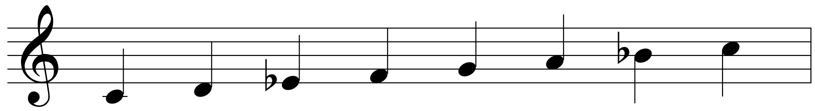
This is the standard “natural” minor scale.

C Locrian



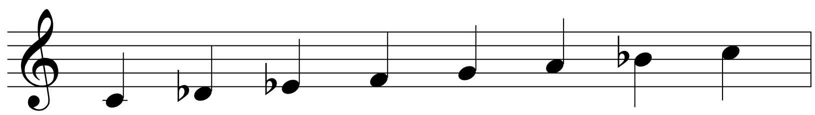
Minor with $\flat\hat{2}$ and $\flat\hat{5}$. Kind of its own thing, really, like a diminished-type scale.

C Dorian



Minor with a raised $\hat{6}$. A fresh and modern-sounding minor!

C Phrygian

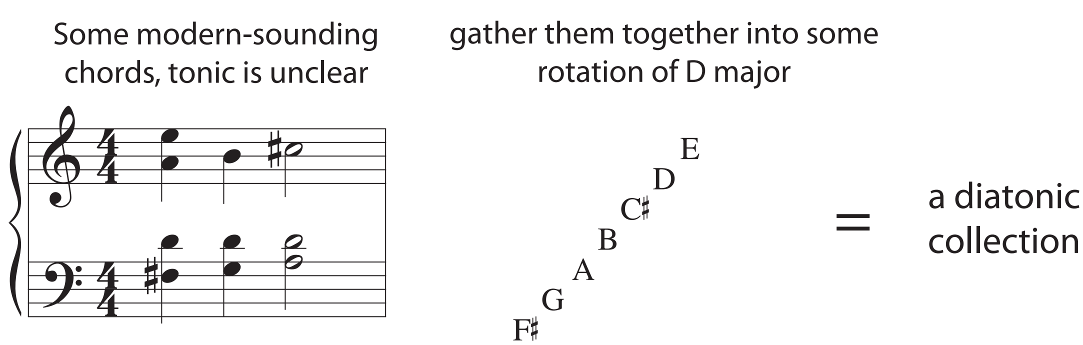


Minor with a $\flat\hat{2}$.

The Diatonic Collection

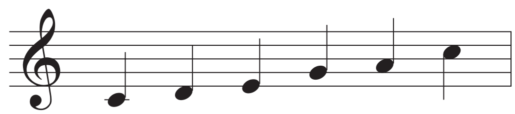
In a sense these modes are all "the same notes," just ordered differently. An E Phrygian scale uses the white keys, organized in such a way that E is the tonic or "home note." F Lydian is the same notes with F as tonic instead.

Sometimes the tones are more loosely organized, so that you can't say for sure where the scale begins and ends. We can talk about this group of notes in general as the *diatonic collection*. A diatonic collection doesn't have to be actual white notes, of course, just any group of tones that can be organized into one of these modes.



The Pentatonic Scale

The pentatonic scale only has five notes in it. It occurs in many different kinds of music throughout the world, and is particularly useful for rock and jazz improvisers. Perhaps the easiest way to remember it would be to build a major triad, fill in the $\hat{2}$, and add a $\hat{6}$, like so:

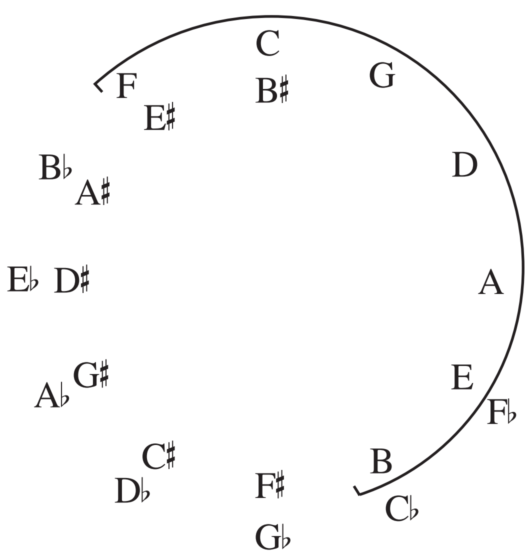


C major? $\hat{1}$ $\hat{2}$ $\hat{3}$ $\hat{5}$ $\hat{6}$

However, the scale can be rotated around other ways to make more shapes. Here is a version that emphasizes the minor triad instead.

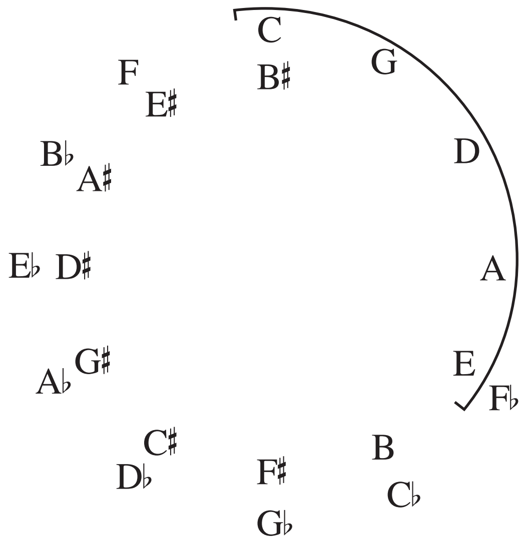
A minor? $\hat{1}$ $\hat{3}$ $\hat{4}$ $\hat{5}$ $\hat{7}$

I think the relationship between the diatonic collection and the pentatonic is very interesting, for two reasons. First, imagine a circle of fifths that contains all possible pitches. (We'll make it curl around on itself to get the different enharmonic spellings.) The C major scale can be generated by taking a segment of this circle. If we shift this zone one position clockwise it generates G major (picking up F#), and if we shift it the other way it makes F major (picking up Bb).



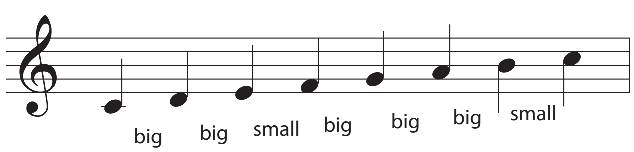
Pitch circle in fifths, C major collection is selected as a 7-note segment.

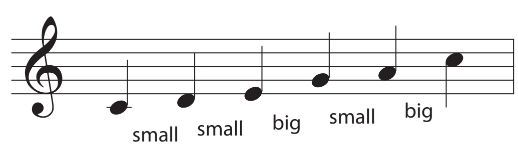
Interestingly, the pentatonic scale is also a smaller segment from this circle, taking five notes instead of seven.



Pentatonic collection is selected as a 5-note segment.

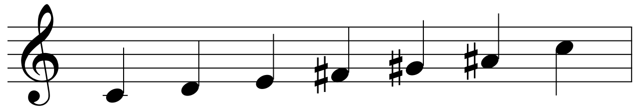
Also, perhaps more perceptually relevant is the way both scales fill up musical space. A diatonic scale fills up the octave with two different intervals - a big one (a whole step) and a smaller one (the half step). The pentatonic also has two interval sizes, but the large is a minor third and the small is a whole step. (It's like the diatonic scale got "swole.")





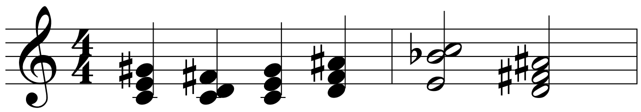
The Whole-Tone Scale

The whole tone scale couldn't be simpler to construct, because it's all whole tones all of the way up.



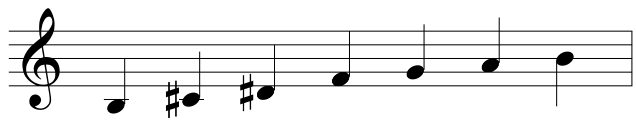
(OK fine, the need to skip one letter name creates the illusion of a gap where there is none, which is a little awkward. Since this is a non-traditional scale there are no fixed rules on how to spell it. Otherwise simple, though!)

This creates an unusual sound, because it can only make augmented triads and other “whole-toney” shapes. (One other familiar chord we might see is the dominant seventh with no fifth in it, like C-D-F#.)



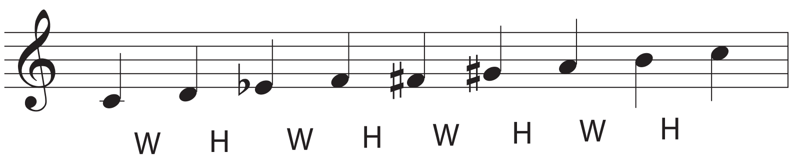
Some whole-tone sonorities

In a sense there are only two whole tone collections. There is the one that might start on C, like we’ve been using so far, and there is the other one that could start on B and contains all of the leftover notes, B-C#-D#-F-G-A.



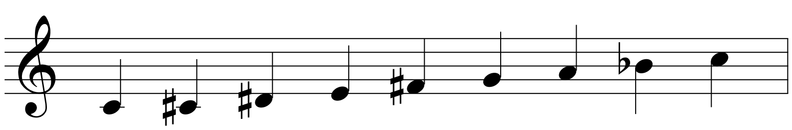
The Octatonic Scale

The octatonic scale alternates whole steps and half steps in a repeating pattern. It manages to cram eight notes into the octave (hence the name “octatonic.”)



octatonic starting on C
(“whole-half” version)

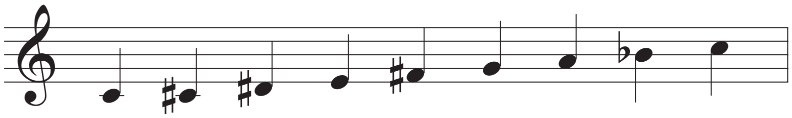
Jazz musicians sometimes call this “the diminished scale.” You could distinguish between two forms of the scale, the “whole-half” one (which might start C-D-Eb, like my example above) and the “half-whole” version (which might start C-C#-D#).



octatonic starting on C
(“half-whole” version)

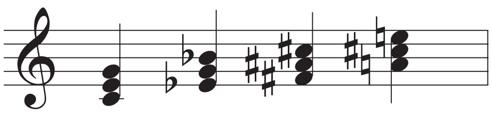
Just as there are a limited number of possible whole-tone scales, there are only three octatonic collections. One somewhat elegant way to specify which one you are talking about would be to say “the octatonic collection that contains C and D,” as opposed to the one that contains C and C# or the one that contains C# and D. Obviously you can use any two adjacent notes in the collection as a reference.

The octatonic scale is particularly useful for composers because it can make a large number of triads and other familiar sonorities.

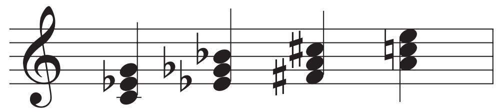


The C-C♯ octatonic collection can produce...

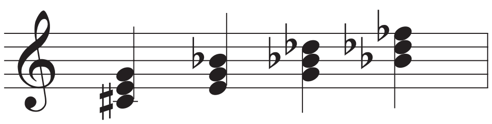
major triads



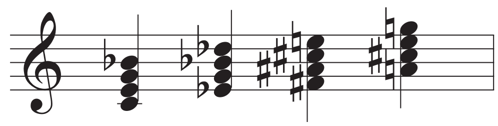
minor triads



diminished triads



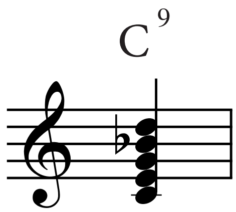
dominant 7th chords

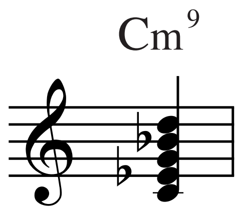


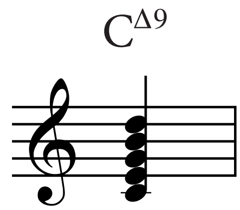
...and much, much more!

Chapter 14: Extended Harmonies (Quick Reference)

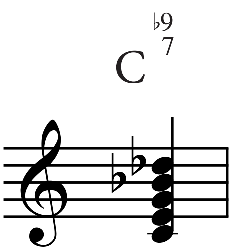
9ths are like a $\hat{2}$ that gets lofted on top of the chord.

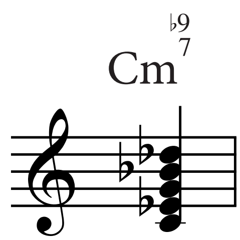






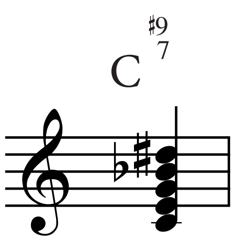
You can modify it to ♭9

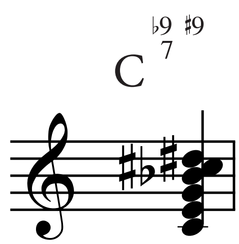




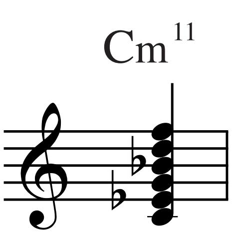
You can modify it to ♯9

You can even mix ♭9 and ♯9

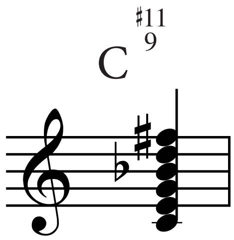


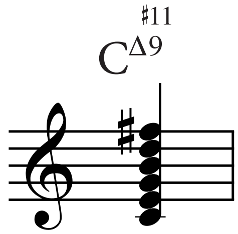


11ths are like a $\hat{4}$ that gets lofted on top of the chord. Implies a 9th as well.

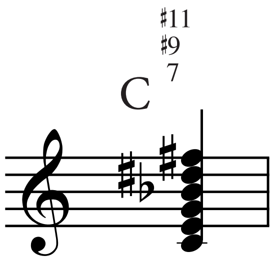


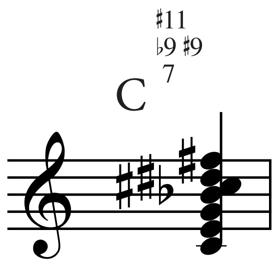
♯11ths are a common modification



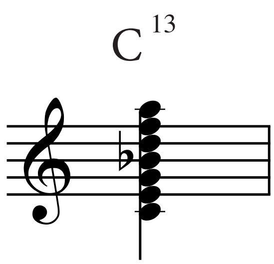


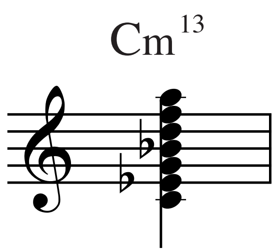
♯11ths mixed with altered 9ths

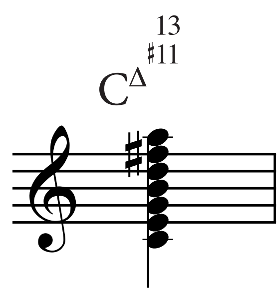


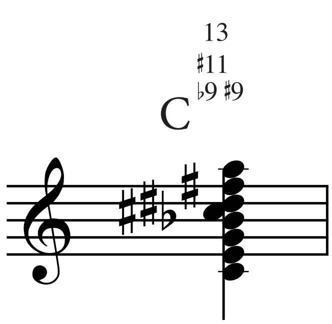


13ths are like a $\hat{6}$ lofted on top of the chord. Implies 9th and 11th.

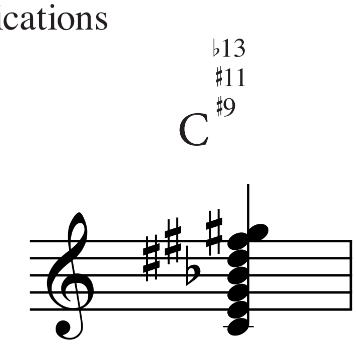








Can be modified as ♭13, combined with other modifications



Chapter 15: Atonal Theory

In the first few decades of the 20th century we started to see music that didn't seem to follow the traditional principles of tonal organization. Composers like Schoenberg, Stravinsky, Webern and Boulez often experimented with unfamiliar, dissonant combinations of notes and intentionally avoided any sense of an underlying scale or tonic note.

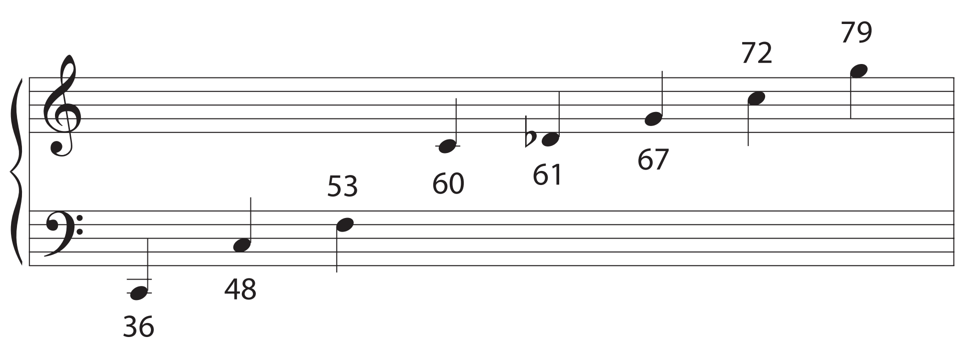
Theorists responded to this new music with a new set of analytic tools, a new "atonal" or "post-tonal" theory. While musical trends have perhaps moved beyond the moment that inspired these ideas, these Modernist concepts can still be stimulating and productive for today's musician.

In this chapter we'll learn a few simple tools that can help us grapple with any combination of notes we might encounter.

Assigning numbers to pitches

Post-Tonal Theory begins by conceptually sweeping away the traditional methods of referring to notes and replacing them with integers. We can still use the traditional note names (like A♭ or C♯) when we feel like it, but we'll understand them as referring to an underlying numerical logic.

So perhaps our first step into this world is to assign a number to every pitch we might encounter on the musical staff. As far as I know there's no standard way to do this in post-tonal theory, but we can conveniently adopt MIDI numbers to refer to our pitches. In the internal computer code that runs MIDI middle C is usually assigned to 60. We can add or subtract twelves to get to C in other octaves.



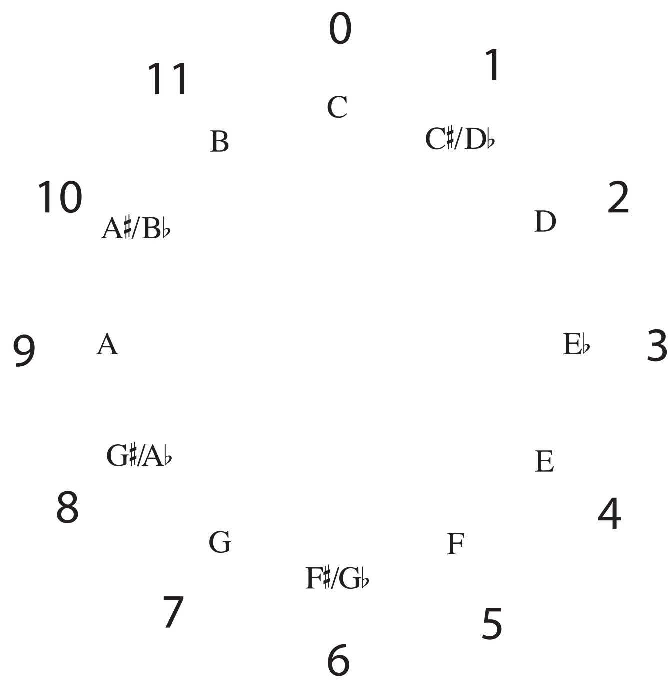
In doing so, we've applied a new rule that transforms the traditional tonal model into our new integer model - we've invoked **enharmonic equivalence**. This is simply the belief that C♯ is "the same note" as D♭, and thus we are assigning them the same MIDI number (i.e. 61 for the C♯/D♭ above middle C.)

Enharmonic equivalence is perhaps not very controversial for the typical theory student, who maybe had a hard time ever believing that C♯ and D♭ were different notes. :)

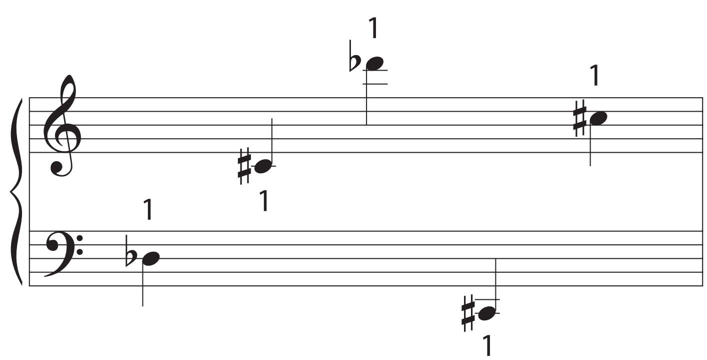
In the discussion that follows I'll continue to use the MIDI numbers whenever we want to refer to actual pitches.

Pitch Classes

To get our **pitch classes** we place all possible notes on a conceptual circle numbered from 0 to 11.



In doing this, we've imposed a new, more aggressive transformation of musical space that can sometimes be counterintuitive. We are arguing that it no longer matters what octave a pitch-class is in - all of the C#'s and Db's in the following example are "the same" pitch-class, 1. This is **octave equivalence**.



Now, in labeling these pitches as PC 1 we are not denying that the notes all sound different. Everybody agrees that they do! But, we are looking for an underlying continuity amongst the notes, and I think we can also agree that they also sound very much like "the same note" in different octaves.

Really, in tonal theory we make octave generalizations as well - we agree that the notes C, E and G represent a C major triad, regardless of where they are distributed on the musical staff. (They can be bunched together or spread out, placed high or low, and rotated around in different ways.) Post-tonal theory looks for the same kind of underlying structure that is provided by chords and scales in traditional theory.

Mod 12

Mathematically, then, we are applying a modulus of 12 to our pitch numbers. Mod 12 math is defined as dividing whatever number you might have by 12 and taking the remainder. So, for example, to determine that middle C is pitch-class 0 we take MIDI Note 60 and divide by 12. The answer is 5 with a remainder of 0, so that's PC 0.

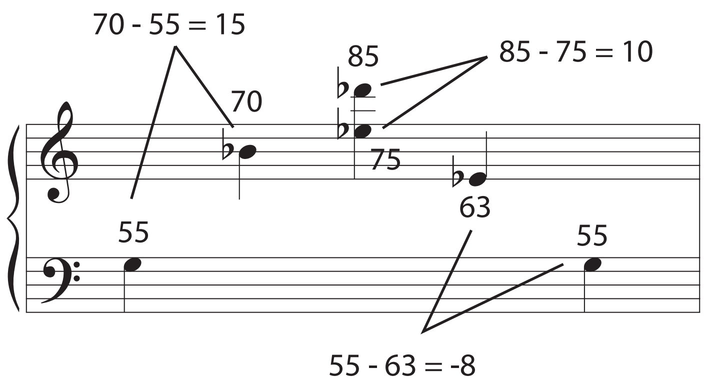
A simpler way to think about it in my opinion is if you find yourself counting higher than 11, subtract 12 until you are in the 0-11 range. If you have a negative number add 12 until you are also in 0-11.

So if we are going to define a pitch class mathmatically we might say that

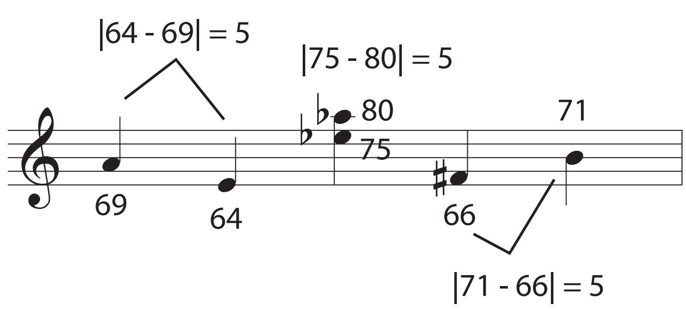
$$\text{pitch class} = \text{mod12(pitch)}$$

Pitch Intervals

Given two pitches a and b, the **directed pitch interval** between them is b-a. (The meaning of "directed" may become clearer as we proceed.) This result will be intuitive if b follows a or is higher than a.



We could also take the absolute value of b-a to account for the counterintuitive cases where a and b seem reversed. If we do that we lose the difference between "up 5" and "down 5," however. We are capturing the pitch interval size but it is no longer "directed."



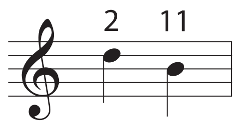
Directed Mod12 Intervals

This is simply the mod12 version of the directed pitch interval, and it's the first thing we can start playing with in our mod12 world.

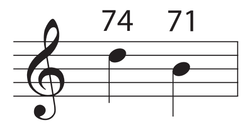
directed mod12 interval = mod12(b-a)

A and b can be two pitch classes or two MIDI pitches.

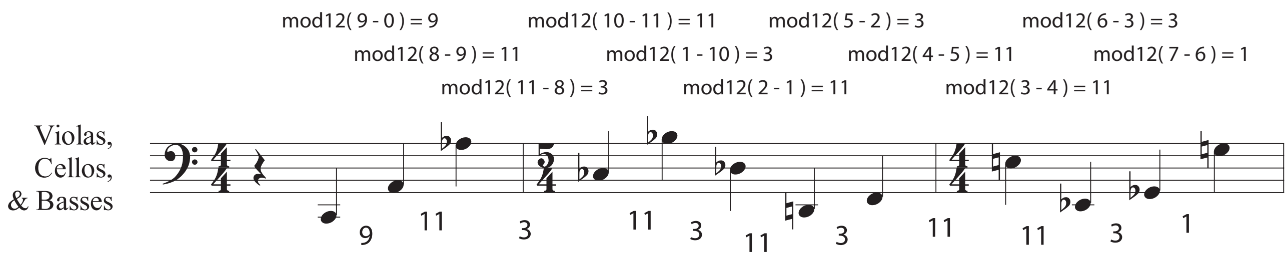
mod12(11 - 2) = 9



mod12(71 - 74) = 9



Let's try it with these notes from Stravinsky's ballet Agon (the "Four Duos" section). I'll base all of the calculations on the pitch classes, since that's faster and easier.



Stravinsky is making a *twelve-tone row* here, a sequence of notes that hits all twelve pitch classes. He will re-use this row as the passage goes on. We can see that he is favoring the mod12 intervals 11 and 3 throughout most of the row - the twisty, octave-displaced nature of the line might conceal this somewhat but once we've done the analysis it is plain as day.

Interval Classes

If we use our directed mod12 intervals we might find 12 possible results, from 0 to 11. Atonal theory also likes to use a higher level of abstraction called the **interval class**. With the interval class we boil down the world of possible intervals into only seven values from 0 to 6.

Here is one way to define the interval class:

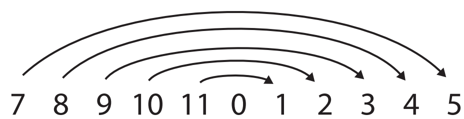
Let x = some interval. To find the IC of x, consider mod12(x) and mod12(-x) and take the lesser of the two.

That looks horrible, but what we are doing is really pretty simple. You may remember that we sometimes practiced our tonal intervals by flipping them around into their "inverted" versions - a major third became a minor sixth, a perfect fourth became a perfect fifth and so on. The interval class takes both versions of the interval and merges them into a single entity.

We can make a little table that tracks some of the traditional tonal intervals that fall into the seven ICs.

IC [0] - all unisons, octaves, multiple octaves.
IC [1] - all minor seconds, major sevenths, and their octave expansions
IC [2] - all major seconds, minor sevenths etc.
IC [3] - minor thirds, major sixths
IC [4] - major thirds, minor sixths
IC [5] - perfect fourths, perfect fifths
IC [6] - the tritone

From a PC interval standpoint, we are identifying intervals 7-11 as inversions of 1-5 and combining them together into a single class.



Pitch-Class Sets and Interval Vectors

For our first pitch-class set, let's select the last chord from Schoenberg's Six Little Pieces for Piano, first movement:

Taking a pitch-class inventory, we've got 10, 4, 8 and 3. We could gather them together and put them in numerical order in a set, like (3, 4, 8, 10). The rounded brackets imply that these PCs are unordered - we reshuffled them when we put together the set, and maybe they'll appear in some other ordering later.

We can explore the sound of this set by deriving an **interval vector**. This is simply an inventory of the intervals in the collection. First we go through every pair of notes in the set, and note the IC made by each one.

(3, 4, 8, 10) [1] (3, 4, 8, 10) [5] (3, 4, 8, 10) [5]

(3, 4, 8, 10) [4] (3, 4, 8, 10) [6]

(3, 4, 8, 10) [2]

Then we gather the information together into a little array, like so:

<1, 1, 0, 1, 2, 1>

This indicates that there is one IC [1], one IC [2] and so on, down the line. With a smattering of almost everything in there this chord is pretty complex.

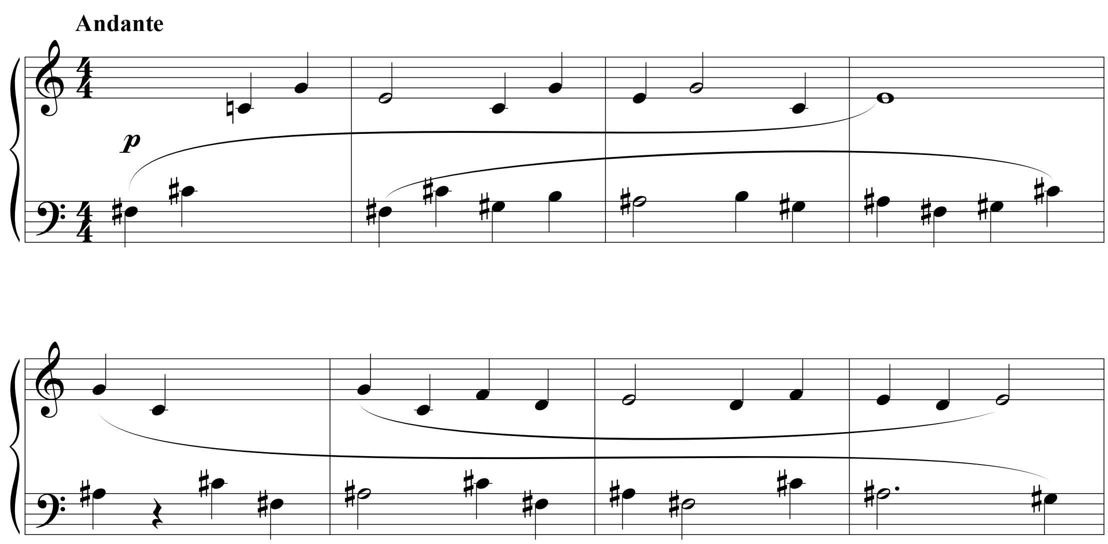
Transposition (T_n)

You are probably familiar with the idea of transposition, which simply moves musical ideas to a new pitch level. In atonal theory transposition is referred to by the function Tn. With Tn you simply decide how much you want to transpose by (that's your n) and you add it a note, set, or melodic segment. We'll consider the result in mod 12.

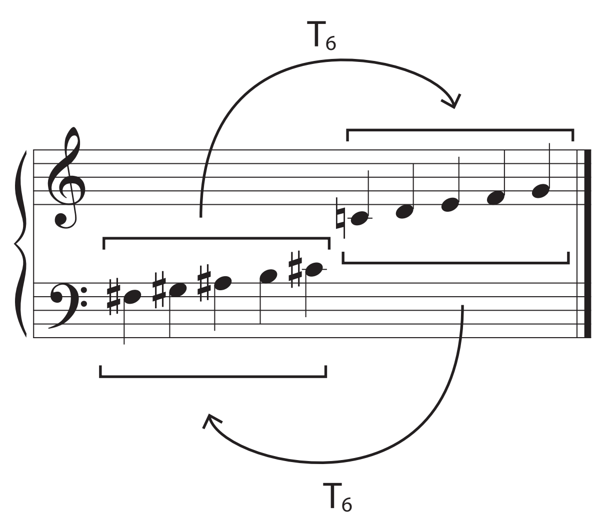
$$T_n(x) = \mathrm{mod}12(x + n)$$

For a simple example of a composer combining together sets related by Tn, let's look at the beginning of this piano piece by Béla Bartók.

Two Major Pentachords, from *Mikrokosmos*



Here the left-hand part uses the first five notes of the F♯-major scale, and the right hand uses the first five notes of C major. The two pentachords are related by T_6. Not only does the left hand's (6, 8, 10, 11, 1) map onto to the right's (0, 2, 4, 5, 7), but the right hand also "wraps around" and transposes back to (6, 8, 10, 11, 1) with another T_6.



Here's the math that transforms one set into the other.

left-hand PCs	T_6	right-hand PCs
1	mod12(1 + 6)	7
11	mod12(11 + 6)	5
10	mod12(10 + 6)	4
8	mod12(8 + 6)	2
6	mod12(6 + 6)	0

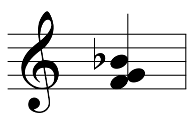
left-hand PCs	T_6	right-hand PCs
7	mod12(7 + 6)	1
5	mod12(5 + 6)	11
4	mod12(4 + 6)	10
2	mod12(2 + 6)	8
0	mod12(0 + 6)	6

Transposition and Inversion (T_nI)

Atonal composers also like to invert things. **Inversion** in post-tonal music turns ideas "upside down," so that the new version is a mirror image of the original. You can do this by multiplying by negative one, and then taking the result mod12.

```
I(x) = mod12(-x)
```

For instance, if we had a three-note chord like so:

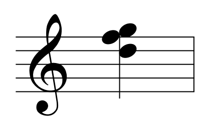


...we could take the set (5, 7, 10) and invert it.

pre-inversion set	multiplied by -1	mod 12 result
10	-10	2
7	-7	5
5	-5	7

Our answer is (2, 5, 7).

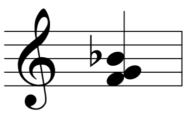
Perhaps we would compose our new chord so that you can see that's it's been flipped around C (pitch-class zero.)



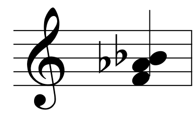
We can bundle the idea of inversion together with transposition in the operation T_nI. Doing both operations means that you could transform a shape into its upside-down version and put it at any pitch level.

$$T_nI(x) = mod12(-x + n)$$

Let's take our sample chord and transform it by T_3I.



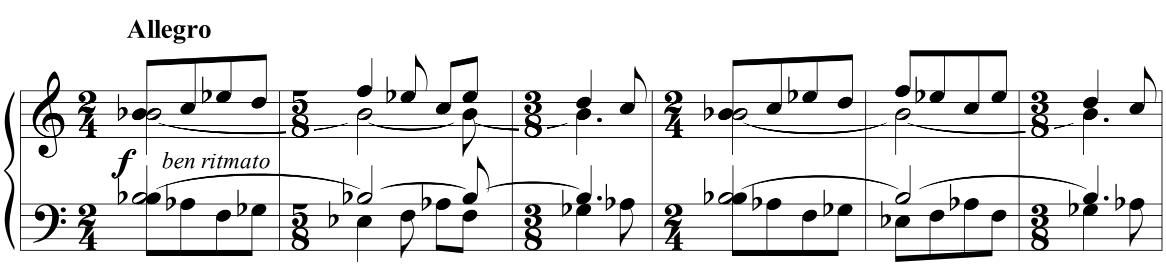
pre-inversion set	multiplied by -1	add 3	mod 12 result
10	-10	-7	5
7	-7	-4	8
5	-5	-2	10



Perhaps you might place the inverted chord in the same pitch area as the original, like I did above.

One interesting thing about inversion is that it creates an **axis of symmetry**. Sometimes this is very obvious in the sheet music, as you can see the shapes swirling around a certain point. In the beginning "Subject and Reflection" Bartok uses B♭ as an axis, and you can see the two parts extending up and down from the held notes in both hands.

Subject and Reflection, from *Mikrokosmos*

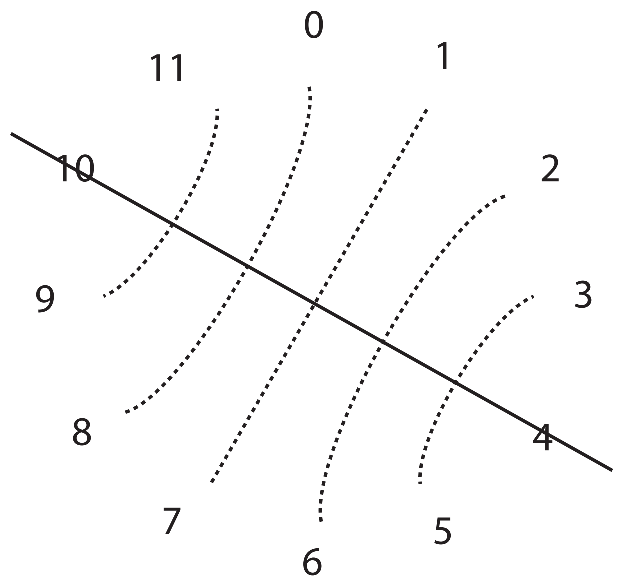


We can transform right hand notes into the left by T_8I.

pre-inversion set	multiplied by -1	add 8	mod 12 result
5	-5	3	3
3	-3	5	5
2	-2	6	6
0	0	8	8
10	-10	-2	10

(It also works the other way - the left maps onto the right with T_8I.)

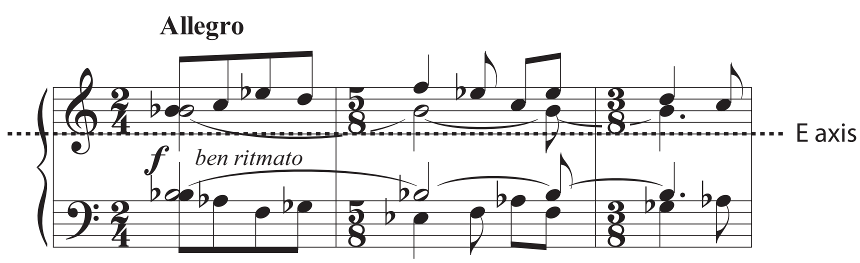
You can map out axes of symmetry on our mod12 circle, showing which notes map onto each other as they undergo a certain T_nI. Here is a map of the Bartok axis around B♭ (pitch-class 10).



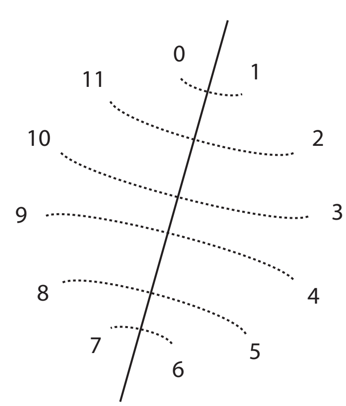
Surprisingly, all symmetrical axes double themselves at the tritone, so you get the same mapping by flipping around PC 4 (E).

Whenever you use T_nI, it makes one of these tritone axes that is based on half of the n. We are using T_8I, so that's our 4 & 10 (E & Bb) axis.

You could argue that E is actually the true axis of the Bartók piece, as it is the pitch that lies exactly in the middle of the two B-flats!



If you use an odd number for n, it creates an axis that is "in the crack" between two notes. T_1I, for instance, creates an axis that falls between PCs 0 and 1 (C and C#) as well as 6 and 7 (F# and G).



Can inversional axes replace traditional keys?

There was a long period in which theorists were very excited by the possibility of inversional axes, thinking that they might actually be a modern substitute for key centers. In my experience I don't think they have the same psychological "pull" of traditional tonics, but they made cool patterns.

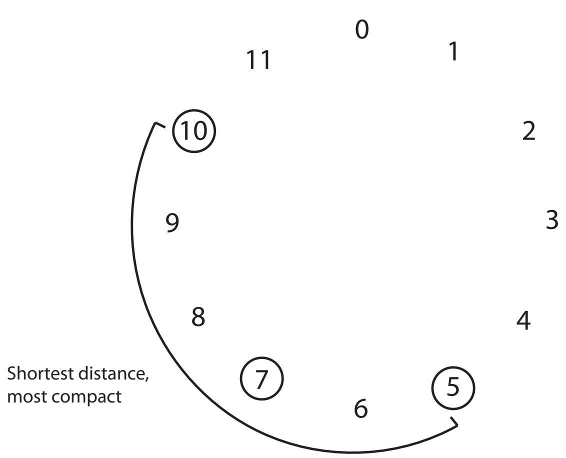
A composer could make their axis a tonic by actually treating it like a tonic - by returning to it frequently and emphasizing it. But I don't think the symmetry contributes to the effect.

Set Types in Prime Form (aka T_n/T_nI types)

All right, this is the main event in atonal theory, a method to take any combination of notes and give it a standardized name. We are going to learn how to take a pitch-class set and boil it down to a preferred version called **prime form**, which functions as a sort of chord name.

We should do this operation on our pitch-class circle. We'll look at the set and look for the version of it that is *most compact* and *most packed to the left*, and we'll transpose that down so that it starts on zero.

First, let's take the three-note chord we were playing with in the previous section, (F, G, B♭).



The interval from 5 to 10 is the shortest span on the circle that includes all the notes.

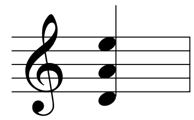
We'll list it starting on 5: (5, 7, 10)

Then we'll transpose down to zero, making a set-type of [0, 2, 5]. That's the standard, referential version of our chord.

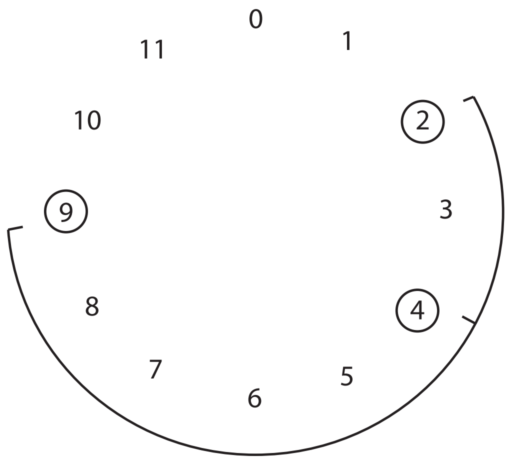
One hint which might come in handy with a more complicated set is that the shortest length on the circle will be the opposite of the largest interval. You can see that everything we *didn't* include in our set is a quite large span of 7 pitch classes. Sometimes it is faster to scan for the biggest interval and take everything but that.

Most packed to the left

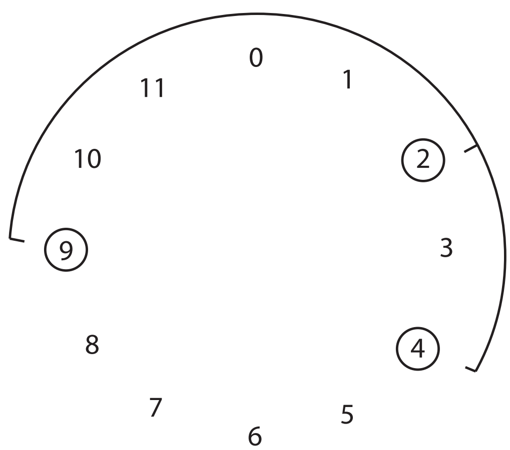
Let's move on to a slightly more complicated example. Let's pick a new, slightly different three-note chord:



On the circle we see that there are two segments with the smallest outside interval.



We could go from 2 to 9, making [0, 2, 7]...



...or we could go from 9 to 4, making [0, 5, 7].

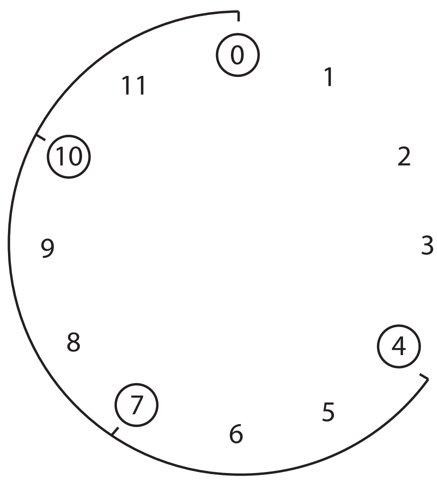
Out of these two possibilities, [0, 2, 7] is **most packed to the left**. The distance from 0 to 2 is less than 0 to 5. So that is our prime form!

(If you are looking at a more complicated set, you might need to compare more than just the first two pitch classes to find the most packed option.)

Considering the inverted shape

There's one more rule, and it's kind of a doozy. We don't just look at the shapes "right-side up" - we also consider the inverted, upside-down version to see if that's more packed to the left.

Let's put a dominant-seventh chord on the circle, (C, E, G, B♭).



Maybe we'll use the "biggest gap" trick to decide that 4-to-0 is the most compact span on the circle that contains all of our PCs.

Starting from 4, we see a potential set-type [0, 3, 6, 8]. However, if we start from zero and go counterclockwise, our result is [0, 2, 5, 8]. That's more packed to the left, so that's our real set-type!

(We could have been more formal about this and actually multiplied our first set by -1, resolved it mod 12, et cetera. However, in my opinion it is usually easy enough to eyeball it by mentally "counting it backwards.")

Final workflow for getting the set type of a chord

1) Think of your notes in numerical order on a mod 12 circle.
2) Find the most compact set (= smallest outside interval) that includes all your PCs. If there is more than one possible version that is the same size list off all of your options.
3) Invert all of your options as well. (Count them backwards.)
4) Choose the version that is the most-packed to the left. Transpose it so it starts on zero.

Of course there are many more "advanced" ideas in atonal theory, but this is the most important one! If you can convert a collection of notes into its set type you have a quick method for identifying and describing it.

List of Set Types

Here is a list of all possible set types in the dodecaphonic system. The labels before each set were introduced by theorist Allen Forte.

2-1 [0, 1]
2-2 [0, 2]
2-3 [0, 3]
2-4 [0, 4]
2-5 [0, 5]
2-6 [0, 6]
3-1 [0, 1, 2]
3-2 [0, 1, 3]
3-3 [0, 1, 4]
3-4 [0, 1, 5]
3-5 [0, 1, 6]
3-6 [0, 2, 4]
3-7 [0, 2, 5]
3-8 [0, 2, 6]
3-9 [0, 2, 7]
3-10 [0, 3, 6]
3-11 [0, 3, 7]
3-12 [0, 4, 8]
4-1 [0, 1, 2, 3]
4-2 [0, 1, 2, 4]
4-3 [0, 1, 3, 4]
4-4 [0, 1, 2, 5]
4-5 [0, 1, 2, 6]
4-6 [0, 1, 2, 7]
4-7 [0, 1, 4, 5]
4-8 [0, 1, 5, 6]
4-9 [0, 1, 6, 7]
4-10 [0, 2, 3, 5]
4-11 [0, 1, 3, 5]
4-12 [0, 2, 3, 6]
4-13 [0, 1, 3, 6]
4-14 [0, 2, 3, 7]
4-Z15 [0, 1, 4, 6]
4-16 [0, 1, 5, 7]
4-17 [0, 3, 4, 7]
4-18 [0, 1, 4, 7]
4-19 [0, 1, 4, 8]
4-20 [0, 1, 5, 8]
4-21 [0, 2, 4, 6]
4-22 [0, 2, 4, 7]
4-23 [0, 2, 5, 7]
4-24 [0, 2, 4, 8]
4-25 [0, 2, 6, 8]
4-26 [0, 3, 5, 8]
4-27 [0, 2, 5, 8]
4-28 [0, 3, 6, 9]
4-Z29 [0, 1, 3, 7]
5-1 [0, 1, 2, 3, 4]
5-2 [0, 1, 2, 3, 5]
5-3 [0, 1, 2, 4, 5]
5-4 [0, 1, 2, 3, 6]
5-5 [0, 1, 2, 3, 7]
5-6 [0, 1, 2, 5, 6]
5-7 [0, 1, 2, 6, 7]
5-8 [0, 2, 3, 4, 6]
5-9 [0, 1, 2, 4, 6]
5-10 [0, 1, 3, 4, 6]
5-11 [0, 2, 3, 4, 7]
5-Z12 [0, 1, 3, 5, 6]
5-13 [0, 1, 2, 4, 8]
5-14 [0, 1, 2, 5, 7]
5-15 [0, 1, 2, 6, 8]
5-16 [0, 1, 3, 4, 7]
5-Z17 [0, 1, 3, 4, 8]
5-Z18 [0, 1, 4, 5, 7]
5-19 [0, 1, 3, 6, 7]
5-20 [0, 1, 5, 6, 8]
5-21 [0, 1, 4, 5, 8]
5-22 [0, 1, 4, 7, 8]
5-23 [0, 2, 3, 5, 7]
5-24 [0, 1, 3, 5, 7]
5-25 [0, 2, 3, 5, 8]
5-26 [0, 2, 4, 5, 8]
5-27 [0, 1, 3, 5, 8]
5-28 [0, 2, 3, 6, 8]
5-29 [0, 1, 3, 6, 8]
5-30 [0, 1, 4, 6, 8]
5-31 [0, 1, 3, 6, 9]
5-32 [0, 1, 4, 6, 9]
5-33 [0, 2, 4, 6, 8]
5-34 [0, 2, 4, 6, 9]
5-35 [0, 2, 4, 7, 9]
5-Z36 [0, 1, 2, 4, 7]
5-Z37 [0, 3, 4, 5, 8]
5-Z38 [0, 1, 2, 5, 8]
6-1 [0, 1, 2, 3, 4, 5]
6-2 [0, 1, 2, 3, 4, 6]
6-Z3 [0, 1, 2, 3, 5, 6]
6-Z4 [0, 1, 2, 4, 5, 6]
6-5 [0, 1, 2, 3, 6, 7]
6-Z6 [0, 1, 2, 5, 6, 7]
6-7 [0, 1, 2, 6, 7, 8]
6-8 [0, 2, 3, 4, 5, 7]
6-9 [0, 1, 2, 3, 5, 7]
6-Z10 [0, 1, 3, 4, 5, 7]
6-Z11 [0, 1, 2, 4, 5, 7]
6-Z12 [0, 1, 2, 4, 6, 7]
6-Z13 [0, 1, 3, 4, 6, 7]
6-14 [0, 1, 3, 4, 5, 8]
6-15 [0, 1, 2, 4, 5, 8]
6-16 [0, 1, 4, 5, 6, 8]
6-Z17 [0, 1, 2, 4, 7, 8]
6-18 [0, 1, 2, 5, 7, 8]
6-Z19 [0, 1, 3, 4, 7, 8]
6-20 [0, 1, 4, 5, 8, 9]
6-21 [0, 2, 3, 4, 6, 8]
6-22 [0, 1, 2, 4, 6, 8]
6-Z23 [0, 2, 3, 5, 6, 8]
6-Z24 [0, 1, 3, 4, 6, 8]
6-Z25 [0, 1, 3, 5, 6, 8]
6-Z26 [0, 1, 3, 5, 7, 8]
6-27 [0, 1, 3, 4, 6, 9]
6-Z28 [0, 1, 3, 5, 6, 9]
6-Z29 [0, 2, 3, 6, 7, 9]
6-30 [0, 1, 3, 6, 7, 9]
6-31 [0, 1, 4, 5, 7, 9]
6-32 [0, 2, 4, 5, 7, 9]
6-33 [0, 2, 3, 5, 7, 9]
6-34 [0, 1, 3, 5, 7, 9]
6-35 [0, 2, 4, 6, 8, 10]
6-Z36 [0, 1, 2, 3, 4, 7]
6-Z37 [0, 1, 2, 3, 4, 8]
6-Z38 [0, 1, 2, 3, 7, 8]
6-Z39 [0, 2, 3, 4, 5, 8]
6-Z40 [0, 1, 2, 3, 5, 8]
6-Z41 [0, 1, 2, 3, 6, 8]
6-Z42 [0, 1, 2, 3, 6, 9]
6-Z43 [0, 1, 2, 5, 6, 8]
6-Z44 [0, 1, 2, 5, 6, 9]
6-Z45 [0, 2, 3, 4, 6, 9]
6-Z46 [0, 1, 2, 4, 6, 9]
6-Z47 [0, 1, 2, 4, 7, 9]
6-Z48 [0, 1, 2, 5, 7, 9]
6-Z49 [0, 1, 3, 4, 7, 9]
6-Z50 [0, 1, 4, 6, 7, 9]
7-1 [0, 1, 2, 3, 4, 5, 6]
7-2 [0, 1, 2, 3, 4, 5, 7]
7-3 [0, 1, 2, 3, 4, 5, 8]
7-4 [0, 1, 2, 3, 4, 6, 7]
7-5 [0, 1, 2, 3, 5, 6, 7]
7-6 [0, 1, 2, 3, 4, 7, 8]
7-7 [0, 1, 2, 3, 6, 7, 8]
7-8 [0, 2, 3, 4, 5, 6, 8]
7-9 [0, 1, 2, 3, 4, 6, 8]
7-10 [0, 1, 2, 3, 4, 6, 9]
7-11 [0, 1, 3, 4, 5, 6, 8]
7-Z12 [0, 1, 2, 3, 4, 7, 9]
7-13 [0, 1, 2, 4, 5, 6, 8]
7-14 [0, 1, 2, 3, 5, 7, 8]
7-15 [0, 1, 2, 4, 6, 7, 8]
7-16 [0, 1, 2, 3, 5, 6, 9]
7-Z17 [0, 1, 2, 4, 5, 6, 9]
7-Z18 [0, 1, 4, 5, 6, 7, 9]
7-19 [0, 1, 2, 3, 6, 7, 9]
7-20 [0, 1, 2, 5, 6, 7, 9]
7-21 [0, 1, 2, 4, 5, 8, 9]
7-22 [0, 1, 2, 5, 6, 8, 9]
7-23 [0, 2, 3, 4, 5, 7, 9]
7-24 [0, 1, 2, 3, 5, 7, 9]
7-25 [0, 2, 3, 4, 6, 7, 9]
7-26 [0, 1, 3, 4, 5, 7, 9]
7-27 [0, 1, 2, 4, 5, 7, 9]
7-28 [0, 1, 3, 5, 6, 7, 9]
7-29 [0, 1, 2, 4, 6, 7, 9]

7-30 [0, 1, 2, 4, 6, 8, 9]
7-31 [0, 1, 3, 4, 6, 7, 9]
7-32 [0, 1, 3, 4, 6, 8, 9]
7-33 [0, 1, 2, 4, 6, 8, 10]
7-34 [0, 1, 3, 4, 6, 8, 10]
7-35 [0, 1, 3, 5, 6, 8, 10]
7-Z36 [0, 1, 2, 3, 5, 6, 8]
7-Z37 [0, 1, 3, 4, 5, 7, 8]
7-Z38 [0, 1, 2, 4, 5, 7, 8]
8-1 [0, 1, 2, 3, 4, 5, 6, 7]
8-2 [0, 1, 2, 3, 4, 5, 6, 8]
8-3 [0, 1, 2, 3, 4, 5, 6, 9]
8-4 [0, 1, 2, 3, 4, 5, 7, 8]
8-5 [0, 1, 2, 3, 4, 6, 7, 8]
8-6 [0, 1, 2, 3, 5, 6, 7, 8]
8-7 [0, 1, 2, 3, 4, 5, 8, 9]
8-8 [0, 1, 2, 3, 4, 7, 8, 9]
8-9 [0, 1, 2, 3, 6, 7, 8, 9]
8-10 [0, 2, 3, 4, 5, 6, 7, 9]
8-11 [0, 1, 2, 3, 4, 5, 7, 9]
8-12 [0, 1, 3, 4, 5, 6, 7, 9]
8-13 [0, 1, 2, 3, 4, 6, 7, 9]
8-14 [0, 1, 2, 4, 5, 6, 7, 9]
8-Z15 [0, 1, 2, 3, 4, 6, 8, 9]
8-16 [0, 1, 2, 3, 5, 7, 8, 9]
8-17 [0, 1, 3, 4, 5, 6, 8, 9]
8-18 [0, 1, 2, 3, 5, 6, 8, 9]
8-19 [0, 1, 2, 4, 5, 6, 8, 9]
8-20 [0, 1, 2, 4, 5, 7, 8, 9]
8-21 [0, 1, 2, 3, 4, 6, 8, 10]
8-22 [0, 1, 2, 3, 5, 6, 8, 10]
8-23 [0, 1, 2, 3, 5, 7, 8, 10]
8-24 [0, 1, 2, 4, 5, 6, 8, 10]
8-25 [0, 1, 2, 4, 6, 7, 8, 10]
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8-27 [0, 1, 2, 4, 5, 7, 8, 10]
8-28 [0, 1, 3, 4, 6, 7, 9, 10]
8-Z29 [0, 1, 2, 3, 5, 6, 7, 9]
9-1 [0, 1, 2, 3, 4, 5, 6, 7, 8]
9-2 [0, 1, 2, 3, 4, 5, 6, 7, 9]
9-3 [0, 1, 2, 3, 4, 5, 6, 8, 9]
9-4 [0, 1, 2, 3, 4, 5, 7, 8, 9]
9-5 [0, 1, 2, 3, 4, 6, 7, 8, 9]
9-6 [0, 1, 2, 3, 4, 5, 6, 8, 10]
9-7 [0, 1, 2, 3, 4, 5, 7, 8, 10]
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12-1 [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

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